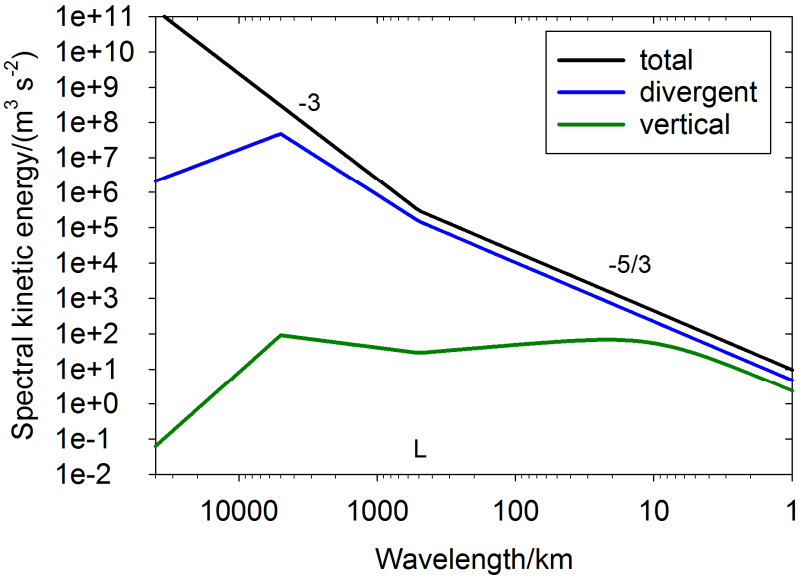


On the spectrum of atmospheric motions

The Horizontal Spectrum of Vertical Velocities near the Tropopause from Global to Gravity Wave and Kolmogorov Scales

Ulrich Schumann

With contributions of Thomas Birner, Martina Bramberger, George Craig, Andreas Dörnbrack, Joseph Egger, Andreas Giez, Sonja Gisinger, Jörgen Jensen, Chris Kruse, Qiang Li, Erik Lindborg, Markus Rapp, Andreas Schäfler, Tobias Selz, Robert Sharman, Nils Wedi,

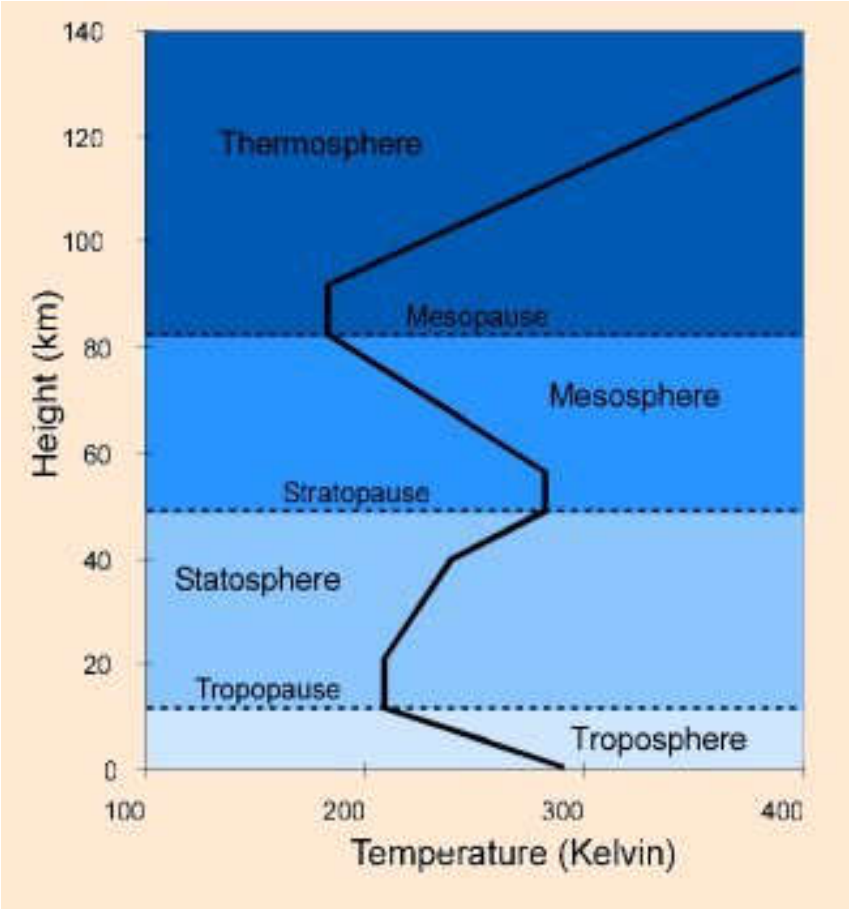
the DEEPWAVE and NAWDEX teams

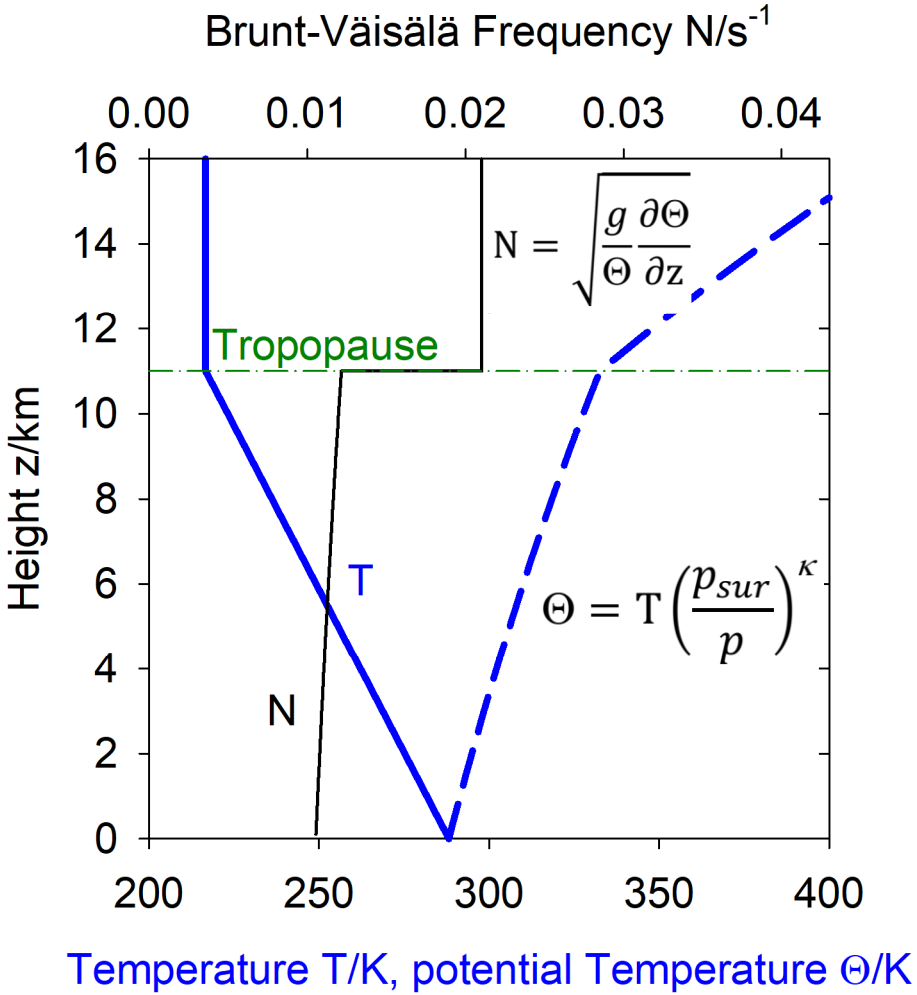
and Michael Eckert and Jessika Wichner

DLR

Atmospheric Dynamics as an application of Fluid Mechanics

The stably stratified atmosphere, below and above the tropopause



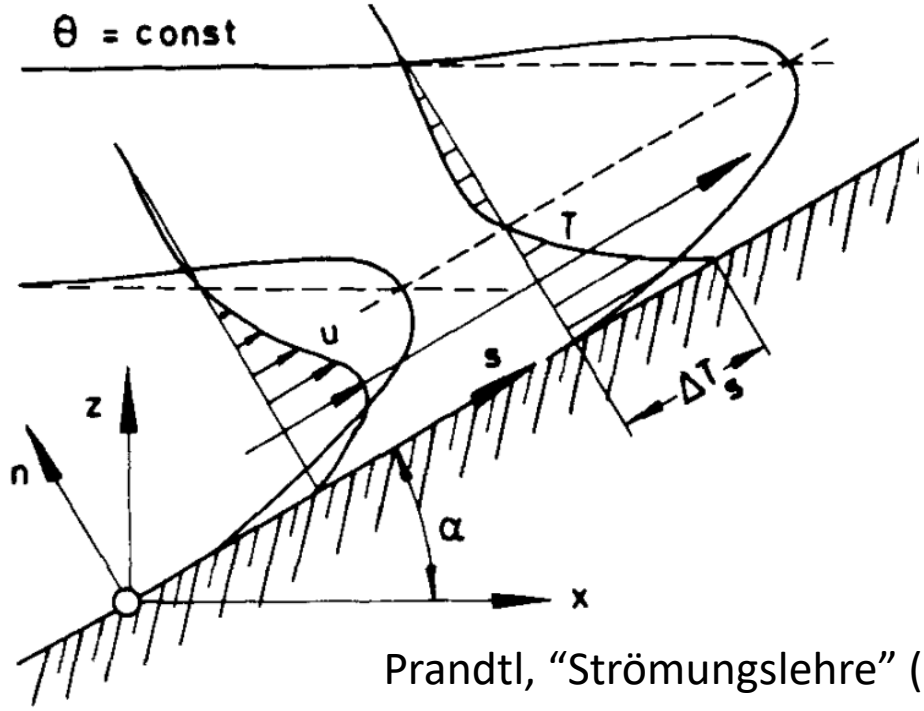


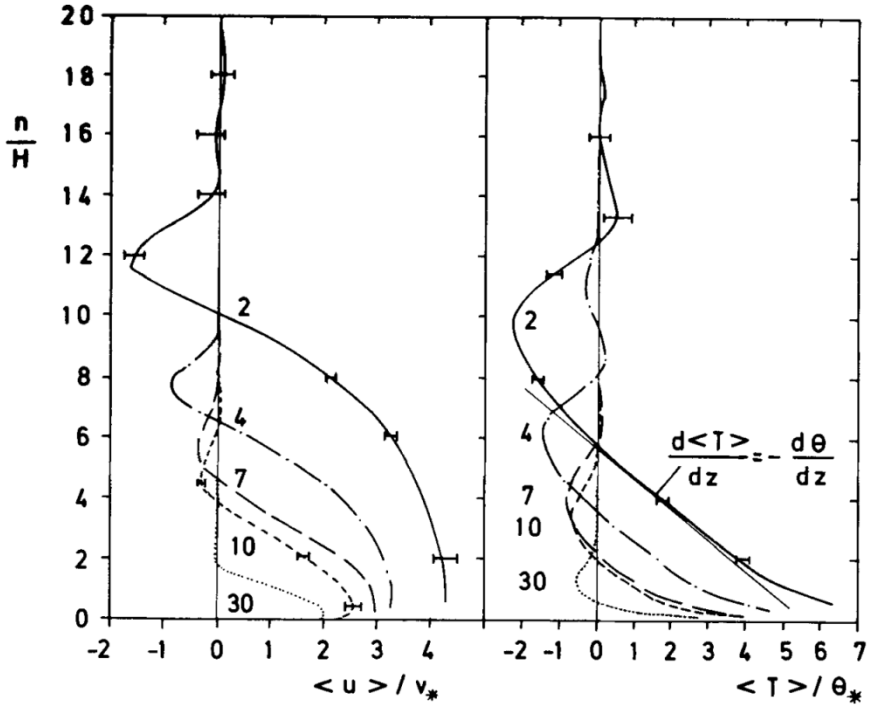
e.g., for ICAO Standard Atmosphere

Ludwig-Prandtl – Memorial Lecture # 63

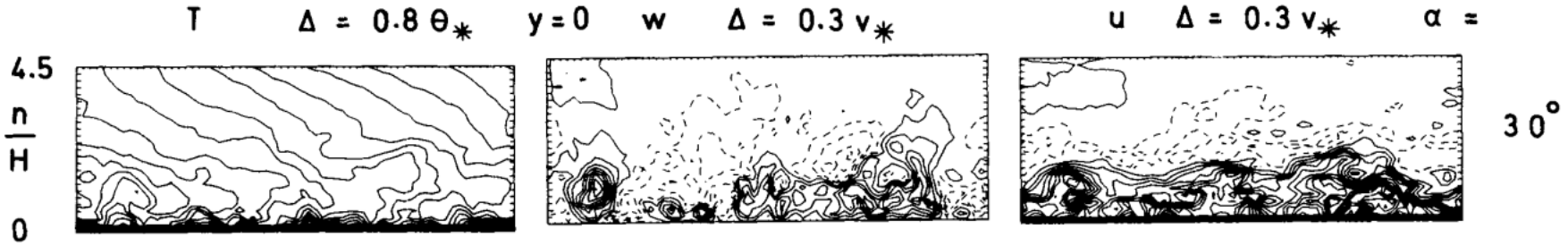
 # 1 (1957): Albert Betz, Göttingen
 # 4 (1969): Ernst Schmidt, TU München
 # 21 (1978): Jürgen Zierep, Karlsruhe
 # 49 (2006): Rainer Friedrich, TU München

LES of Prandtl's slope boundary layer



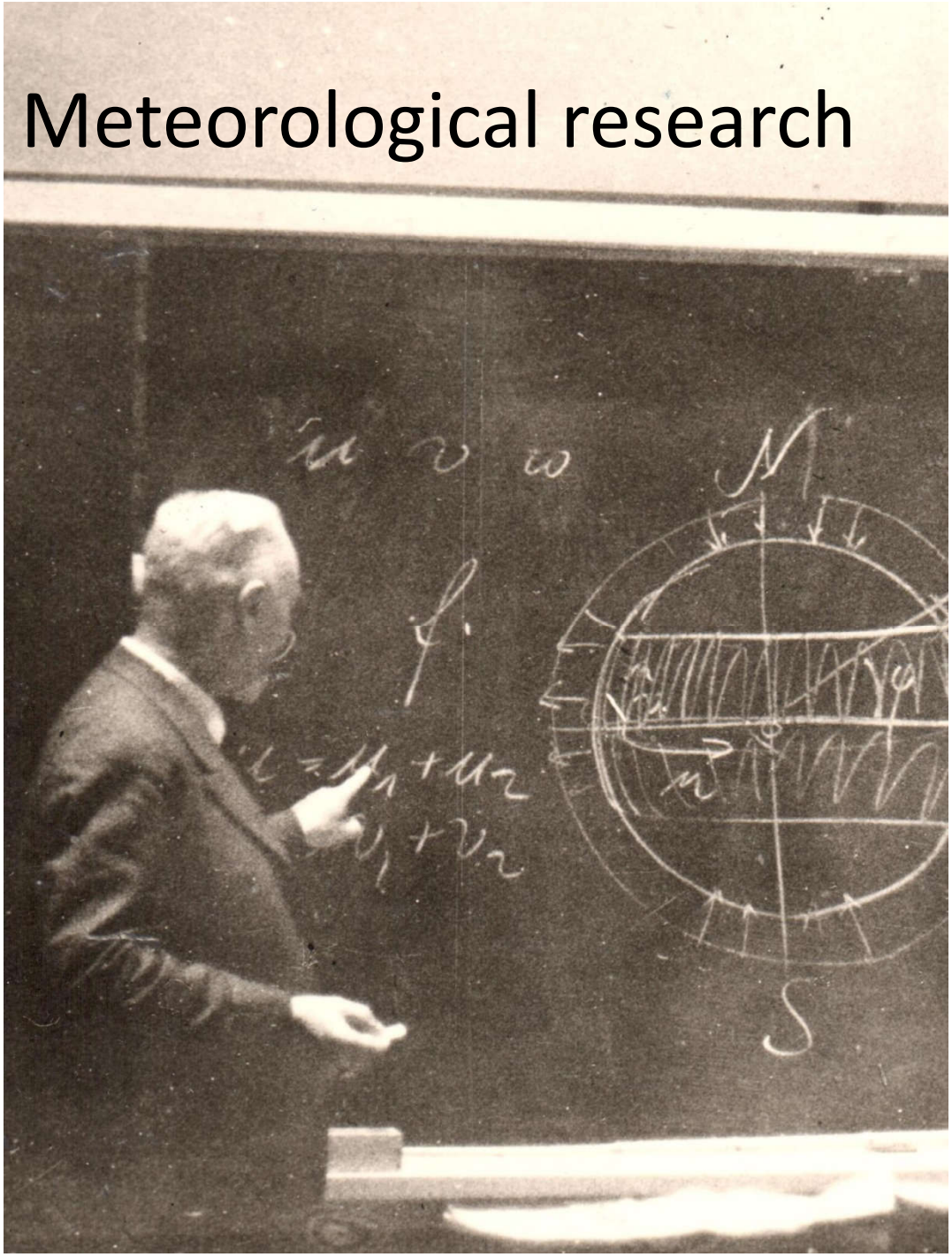


Prandtl, "Strömungslehre" (1942)



Schumann, Quart. J. Roy. Met. Soc. (1990)

Meteorological research



"a suitable occupation during peace"

(1945, cited from Eckert, 2019)

Turbulent Boundary Layer

Turbulent kinetic energy

Slope wind

Rotating flows, $f=2\,\omega \sin(\varphi)$

Vorticity conservation

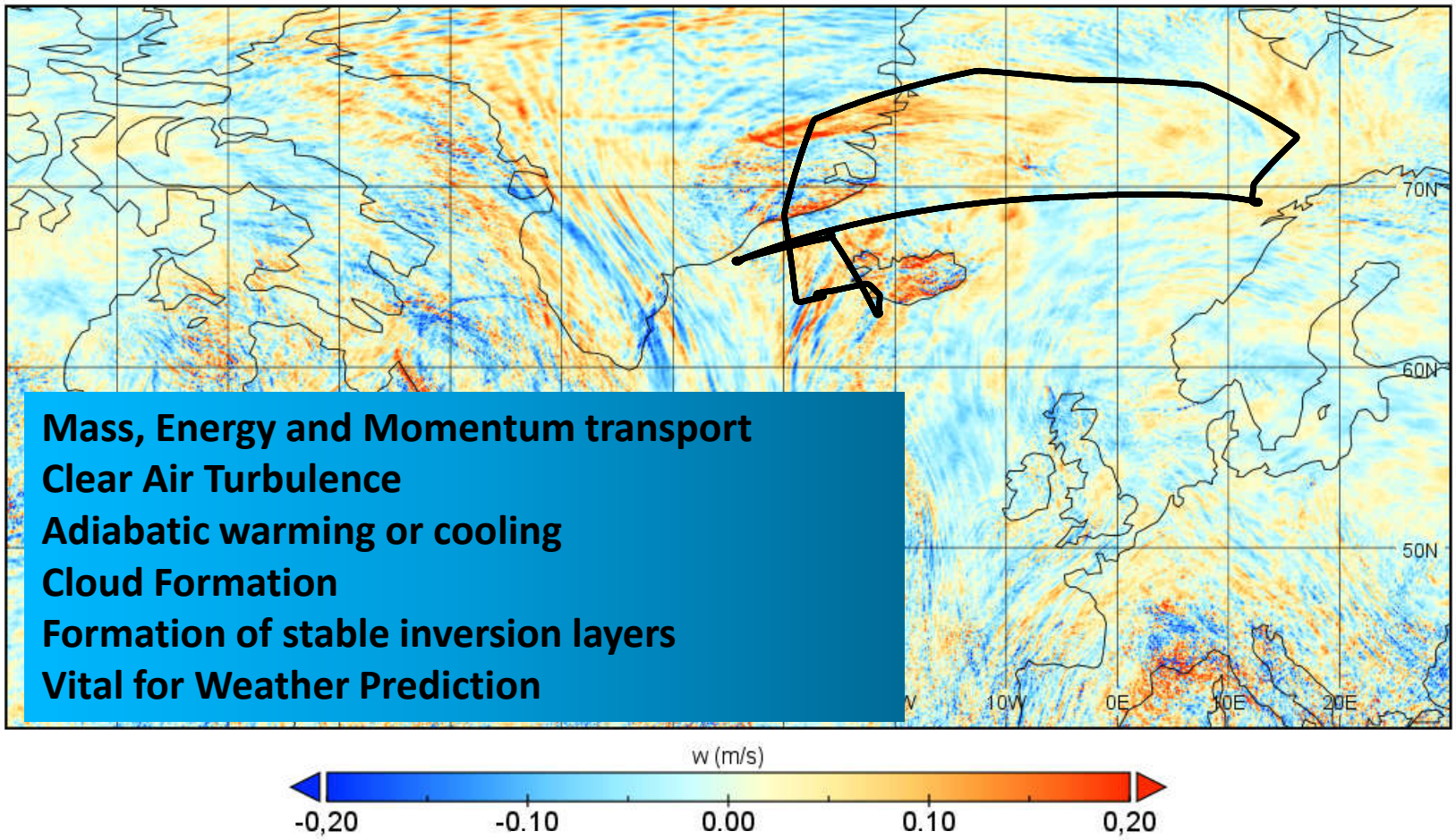
Weather Prediction

Vertical velocity (w) – Fundamental for Atmospheric Dynamics

p=200 hPa (≈ 12 km), 12 UTC 13 Oct 2016

HALO during NAWDEX

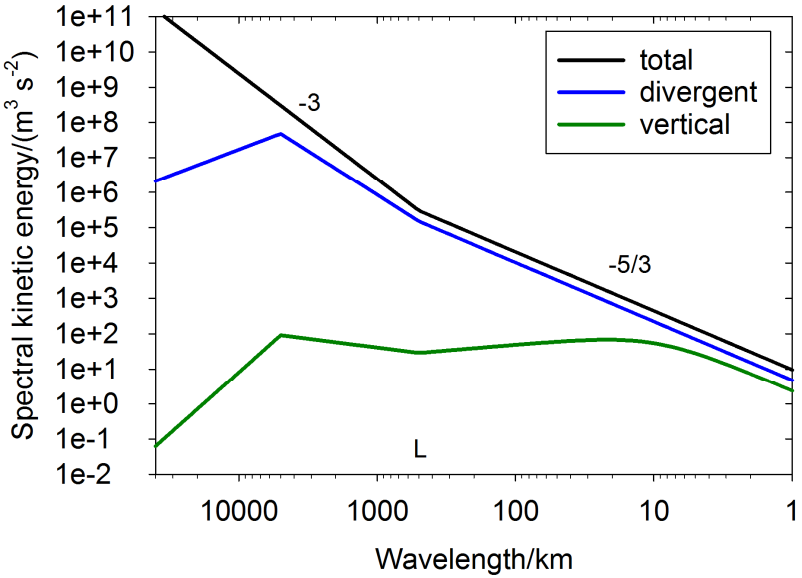
w



min=-1.3 m/s, max=1.5 m/s

ECMWF-IFS TCo7999 (Nils Wedi), 1.2 km grid resolution

10 to 100 times smaller than horizontal wind



1. What do we know about spectra of horizontal velocities (h-spectrum)
2. What do we know about spectra of vertical velocities (w-spectrum)
3. A hypothesis relating w-spectra with h-spectra
4. Validation
5. Implications
6. Conclusions

DLR

What do we know about spectra of horizontal velocities:
from long-distance commercial aircraft data

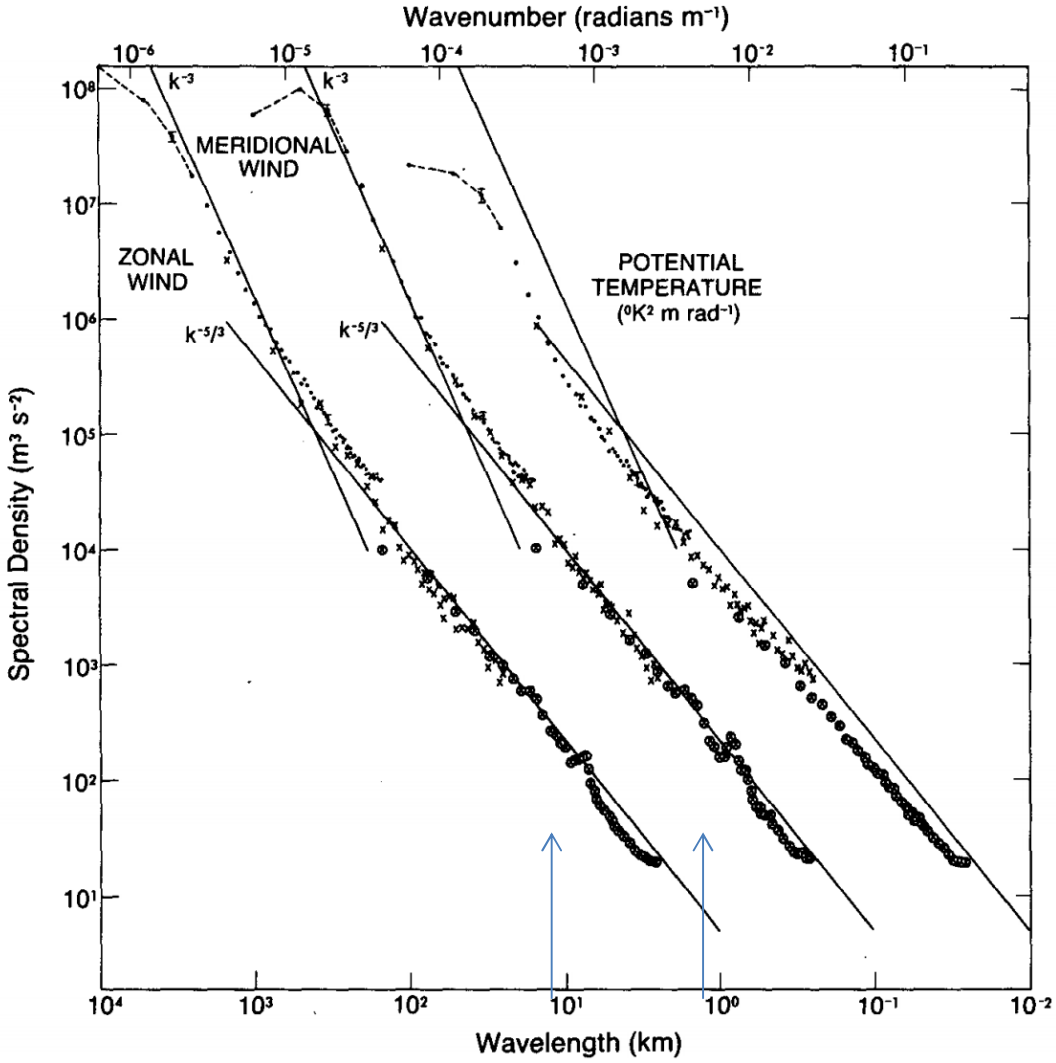


FIG. 3. Variance power spectra of wind and potential temperature near the tropopause from GASP aircraft data. The spectra for meridional wind and temperature are shifted one and two decades to the right, respectively; lines with slopes −3 and $-\frac{5}{3}$ are entered at the same relative coordinates for each variable for comparison.

Nastrom & Gage (1985)

GASP: Global Atmospheric Sampling Program

6000 flights
B747 airliners,
Prandtl's pitot pipe (TAS) and heading,
9–13 km altitude
at horizontal scales of 2.6 to 10000 km

The "Canonical Spectrum" (Skamarock et al., 2014) of horizontal velocities from long-distance commercial aircraft data

(GASP: Nastrom & Gage, 1985; MOZAIC: Lindborg, 1999)

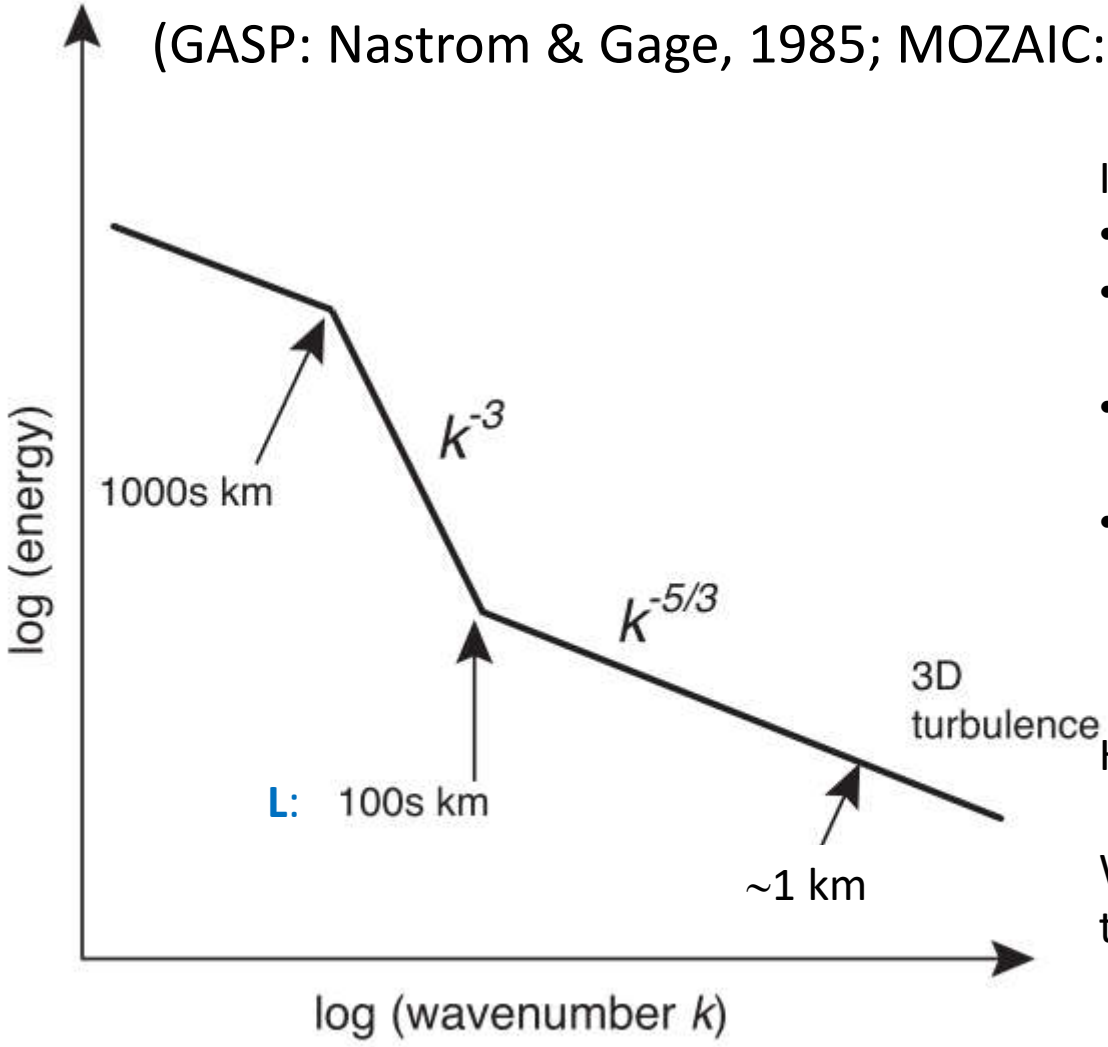


FIG. 1. Schematic of canonical atmospheric kinetic energy spectra.

Important for:

- Understanding
- Model tests (Skamarock et al., 2014; Wedi 2016)
- NWP predictability (Lorenz, 1969)
- SGS modeling

How to explain?

What determines the transition scale L?

Large-scale k^{-3} spectrum: Quasi-geostrophic turbulence

(Charney, 1971)

$$\boldsymbol{f} \times \boldsymbol{u} \approx -\frac{1}{\rho}\nabla_z p,$$

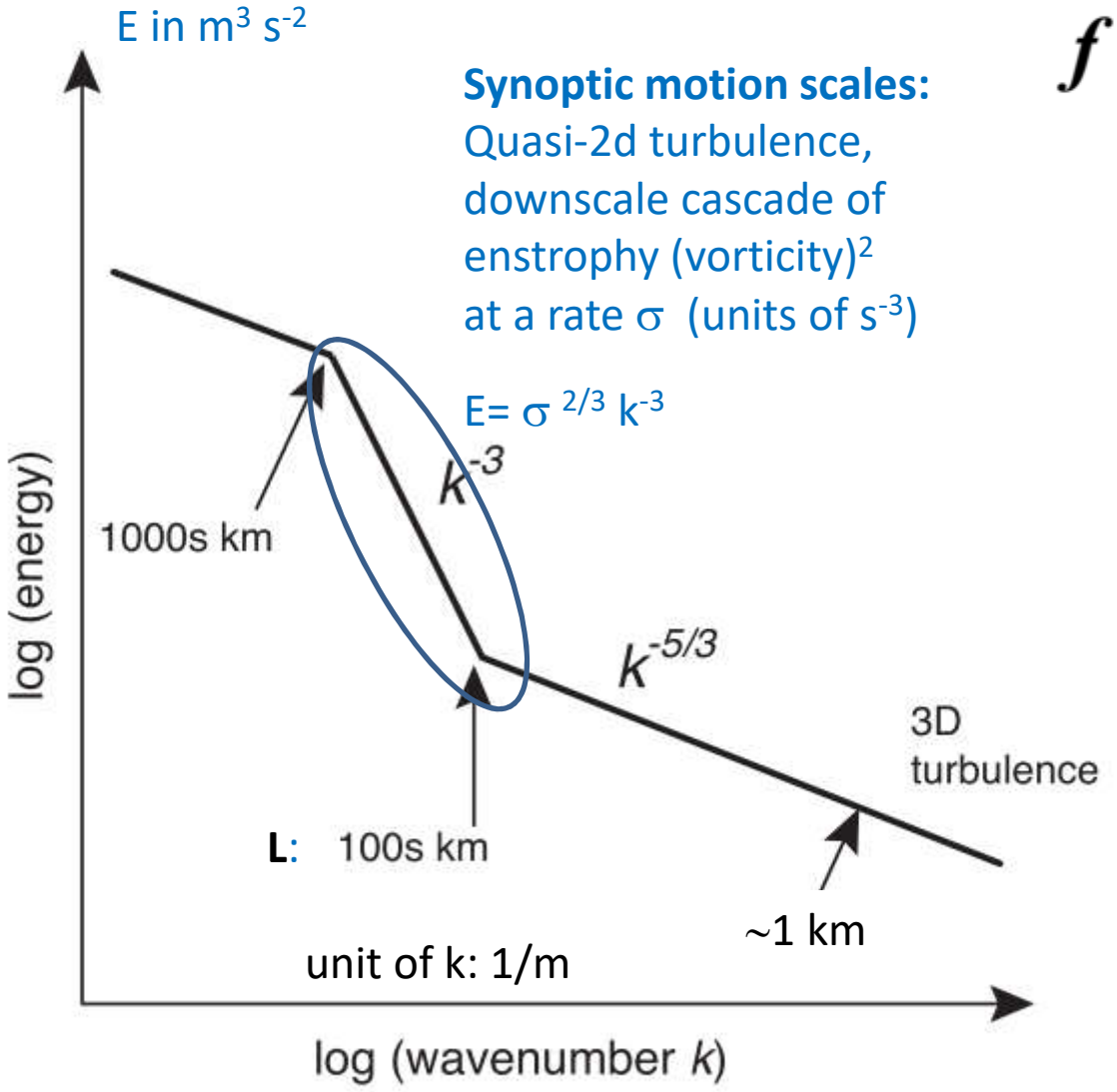


FIG. 1. Schematic of canonical atmospheric kinetic energy spectra.

Small-scale $k^{-5/3}$ spectrum: Inertial-range energy cascade
(Kolmogorov, 1941)

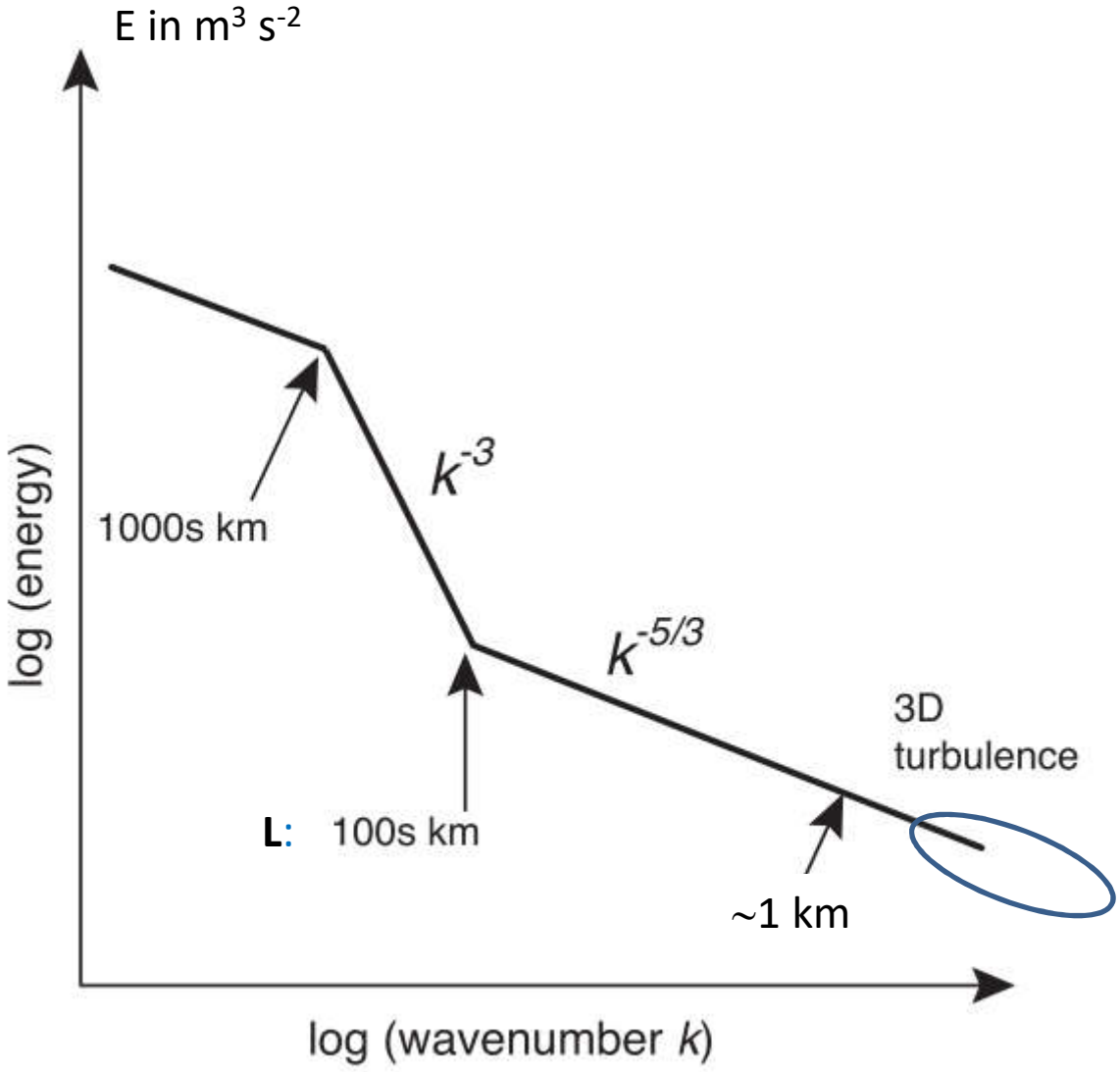
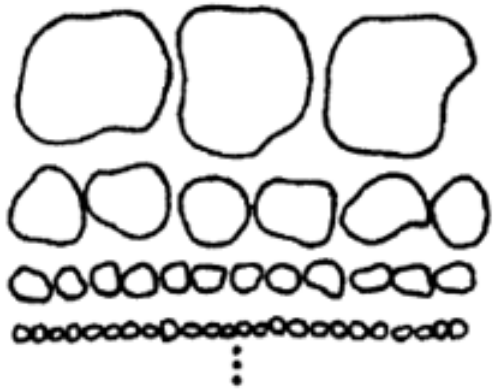


FIG. 1. Schematic of canonical atmospheric kinetic energy spectra.



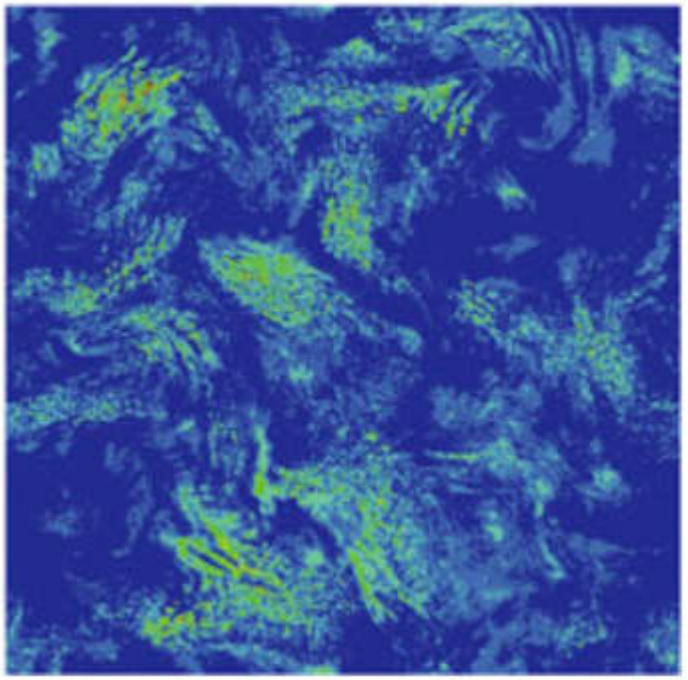
Richardson Model

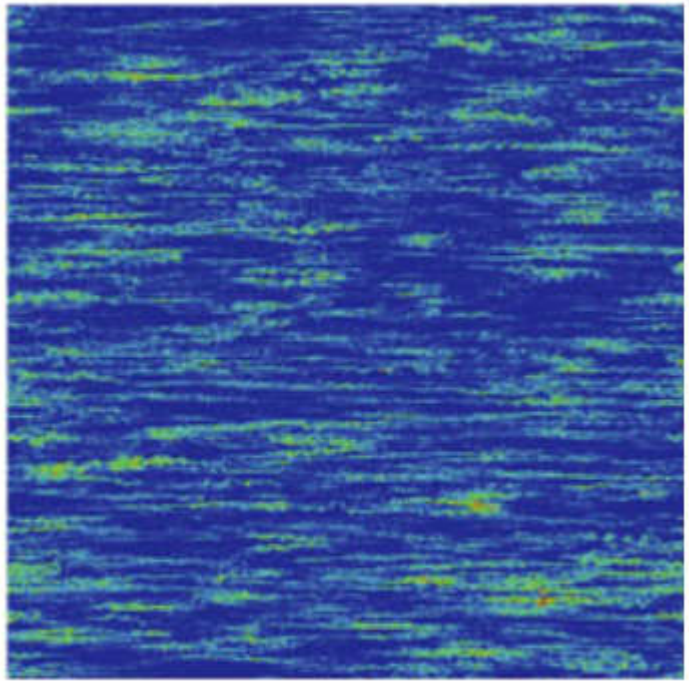
Turbulence scales:
Downscale energy transfer with energy dissipation rate ε (unit: $m^2\ s^{-3}$)

$E = \varepsilon^{2/3} k^{-5/3}$

Basic, e.g., for LES SGS

Stably stratified turbulence





DNS, 1024^3 grid cells,
homogeneous, stratified turbulence,
forced at large scales
(Kimura and Herring, JFM, 2012)

Stably stratified turbulence is strongly anisotropic for scales > Obukhov scale
$L_O = (\varepsilon/N^3)^{1/2}$

Mesoscale $k^{-5/3}$ spectrum: stably stratified turbulence

(e.g., Lilly, 1983; Riley and Lindborg, 2008)

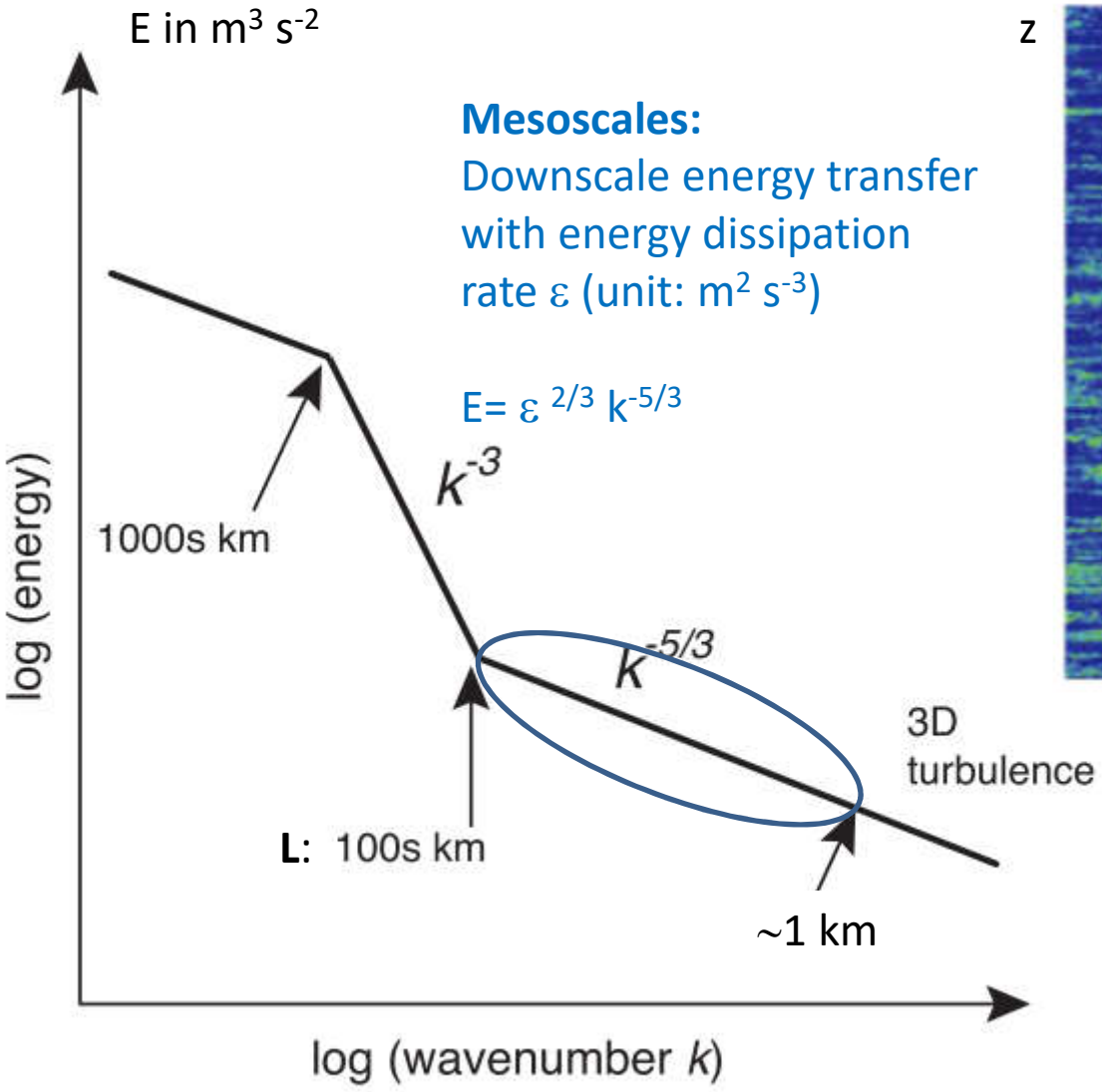
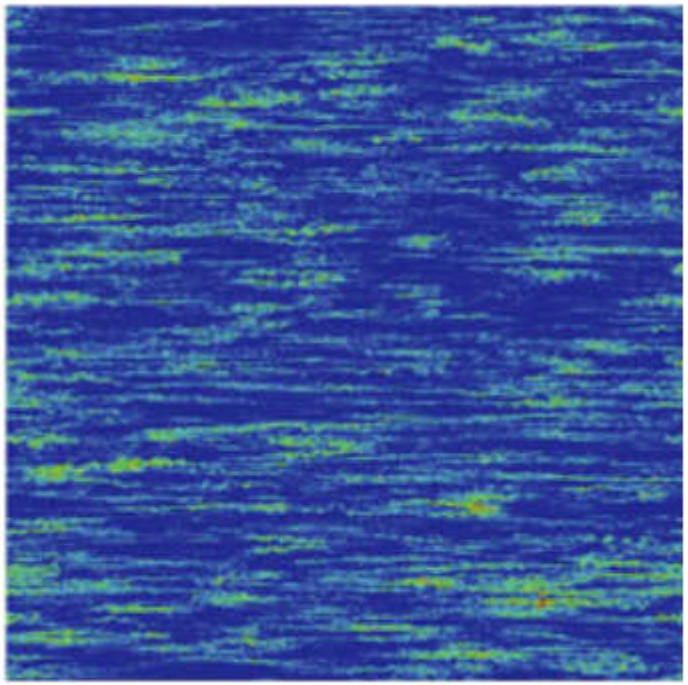


FIG. 1. Schematic of canonical atmospheric kinetic energy spectra.

Mesoscales:
Downscale energy transfer with energy dissipation rate ε (unit: $m^2\ s^{-3}$)

$E = \varepsilon^{2/3} k^{-5/3}$

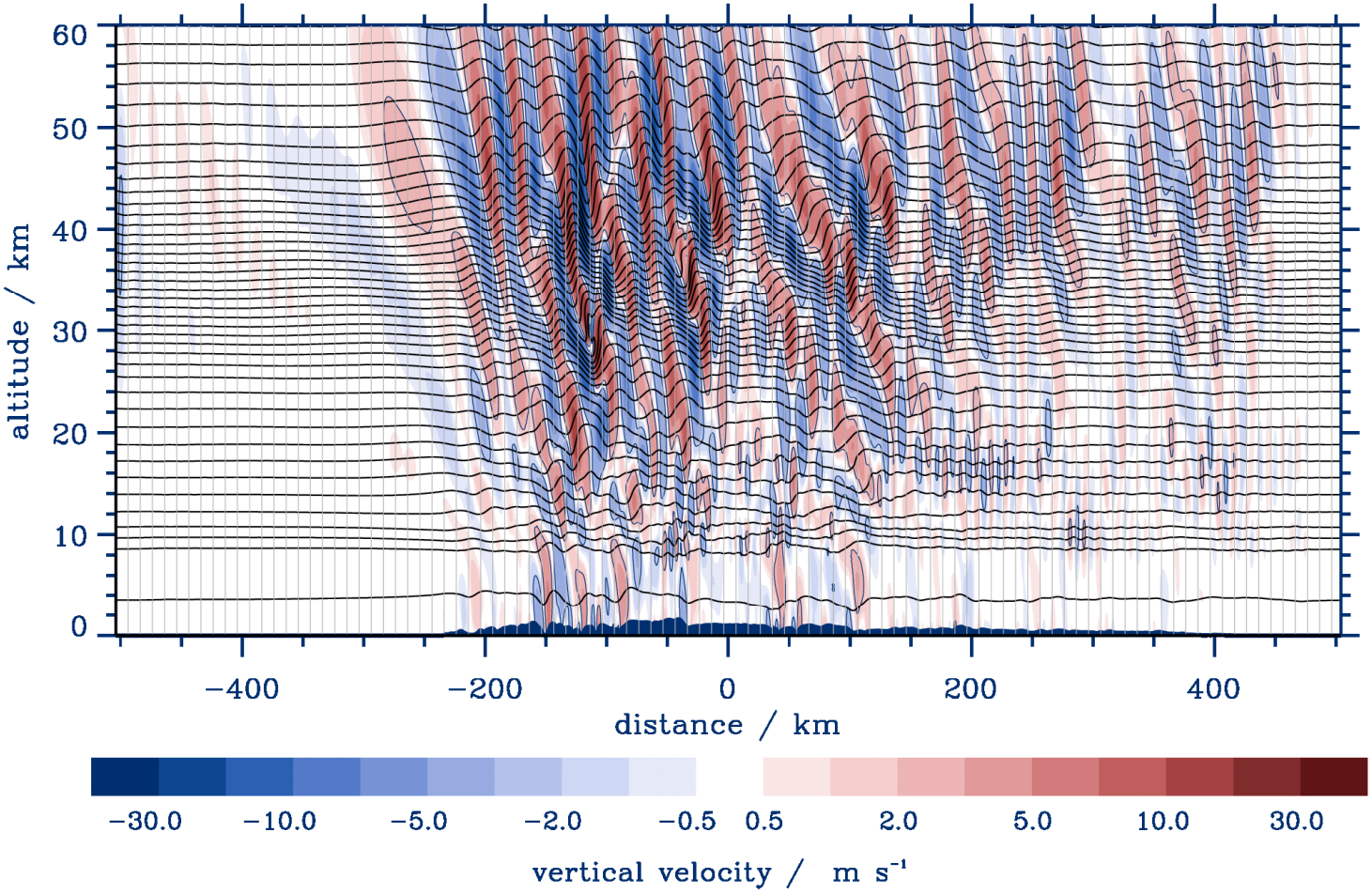


Downscale energy transfer rate:
$\varepsilon = u'^3/L$

u' = horizontal velocity fluctuation
L = horizontal length scale.

Gravity waves in stratosphere over mountains

(vertically upward energy transfer, not downscale)



Waves: disturbances which travel over long distances and transport energy and momentum

Dörnbrack (2019).

Helmholtz decomposition (1858) for horizontal velocity

Hermann von Helmholtz (1821-1894)

$$\boldsymbol{v} = \boldsymbol{\nabla} \times \boldsymbol{\psi} + \boldsymbol{\nabla}\chi$$

(Helmholtz, Crelle J., 1858)

For horizontal velocity $\boldsymbol{v} = (u, v)$ on a smooth surface with normal $\boldsymbol{k}$:

$$\boldsymbol{v} = \boldsymbol{k} \times \boldsymbol{\nabla}\psi + \boldsymbol{\nabla}\chi$$

$\boldsymbol{v} = \boldsymbol{v_R} + \boldsymbol{v_D}, \qquad \boldsymbol{v_R} = \boldsymbol{k} \times \boldsymbol{\nabla}\psi, \qquad \boldsymbol{v_D} = \boldsymbol{\nabla}\chi.$ (Wippermann, Beitr. Phys. Atm., 1957)

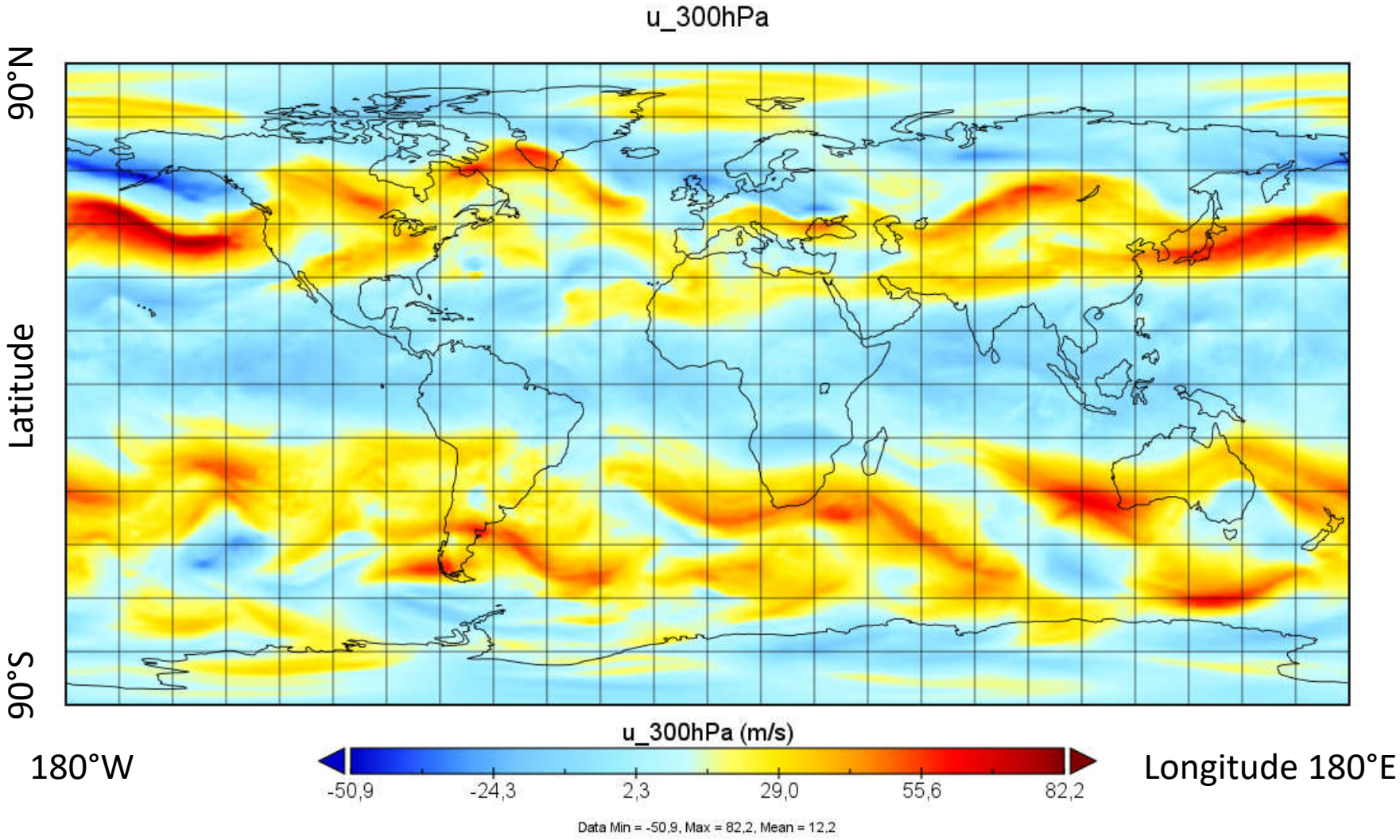
The stream function and velocity potential follow from solutions of Poisson equations

$$\nabla^2 \psi = \zeta = \boldsymbol{k} \cdot (\boldsymbol{\nabla} \times \boldsymbol{v})$$

$$\nabla^2 \chi = \delta = \boldsymbol{\nabla} \cdot \boldsymbol{v}$$

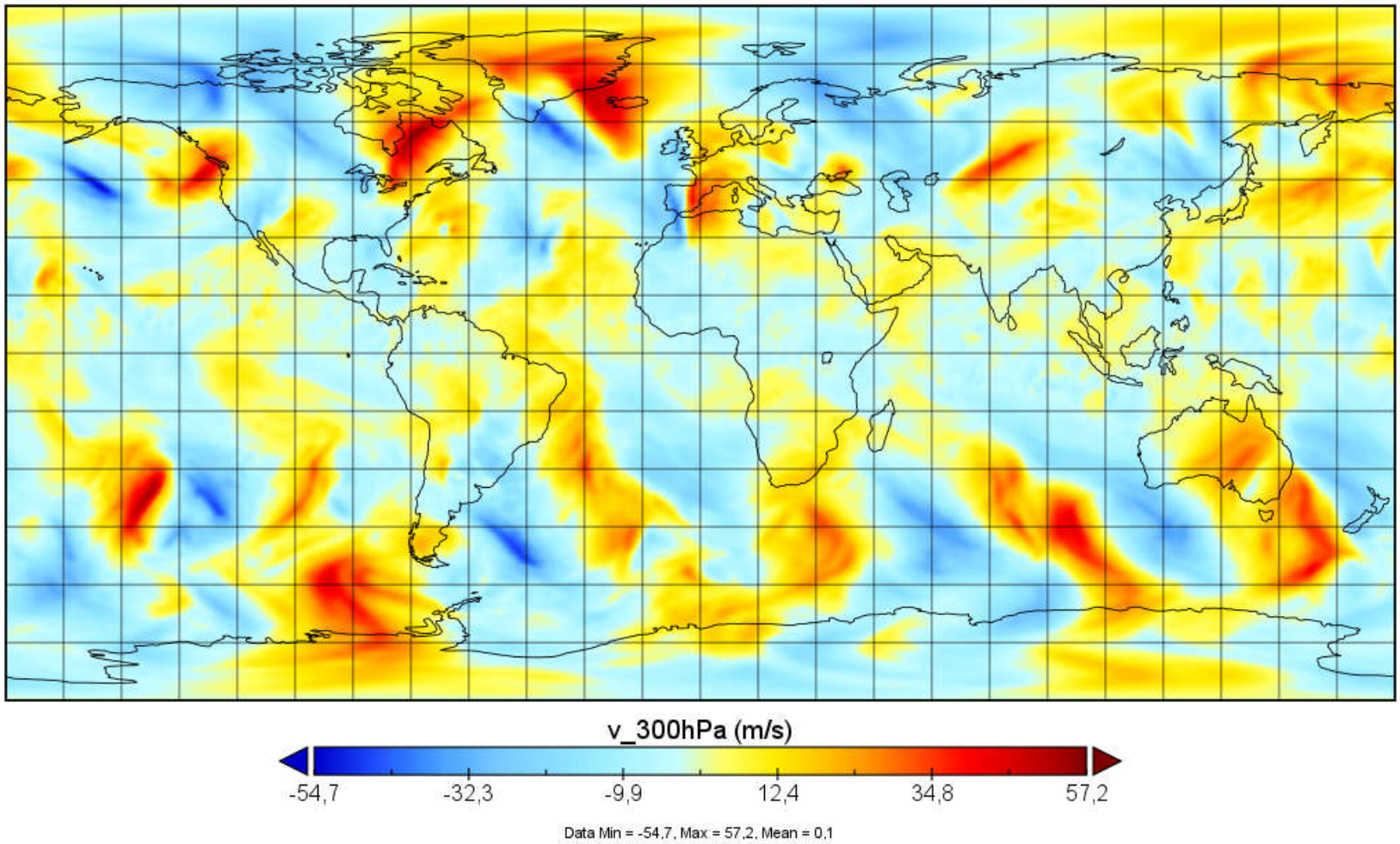
Illustration of Helmholtz decomposition

u, v: horizontal wind components in east and north directions



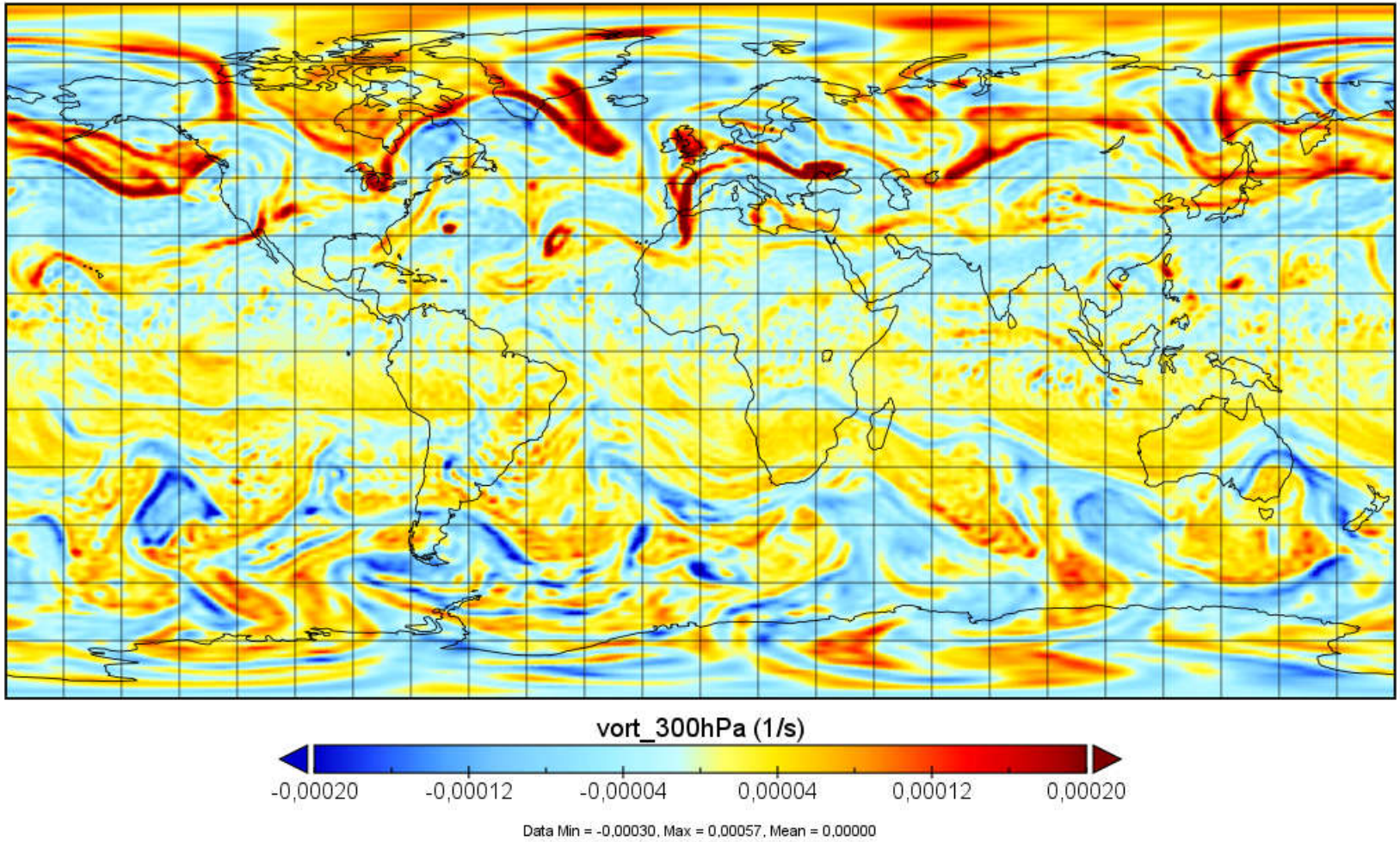
NAWDEX case: 12 UTC 13 Oct 2016, p= 300 hPa ≈ 9 km height

v_300hPa



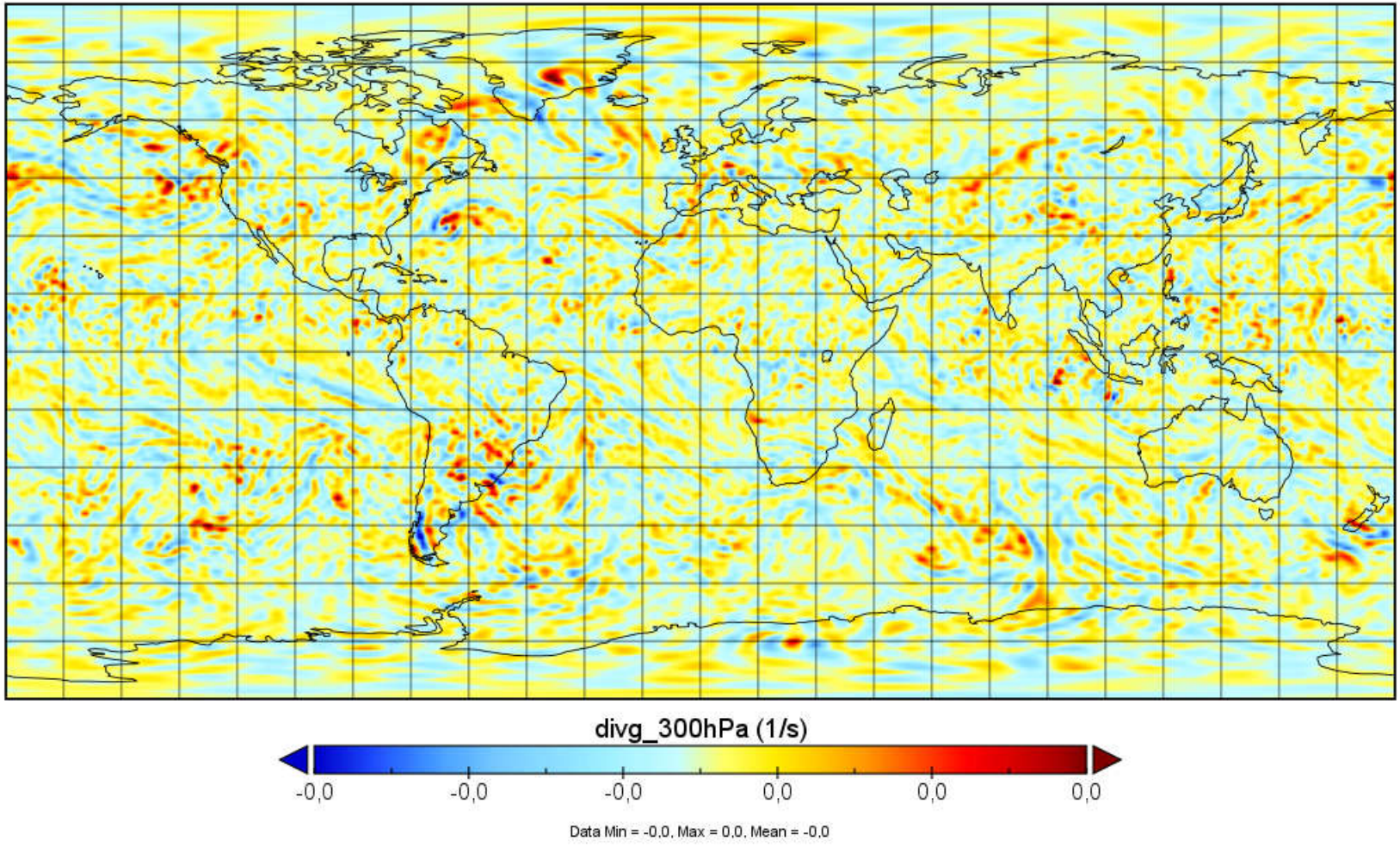
$$\zeta = \boldsymbol{k} \cdot (\boldsymbol{\nabla} \times \boldsymbol{v})$$

vort_300hPa



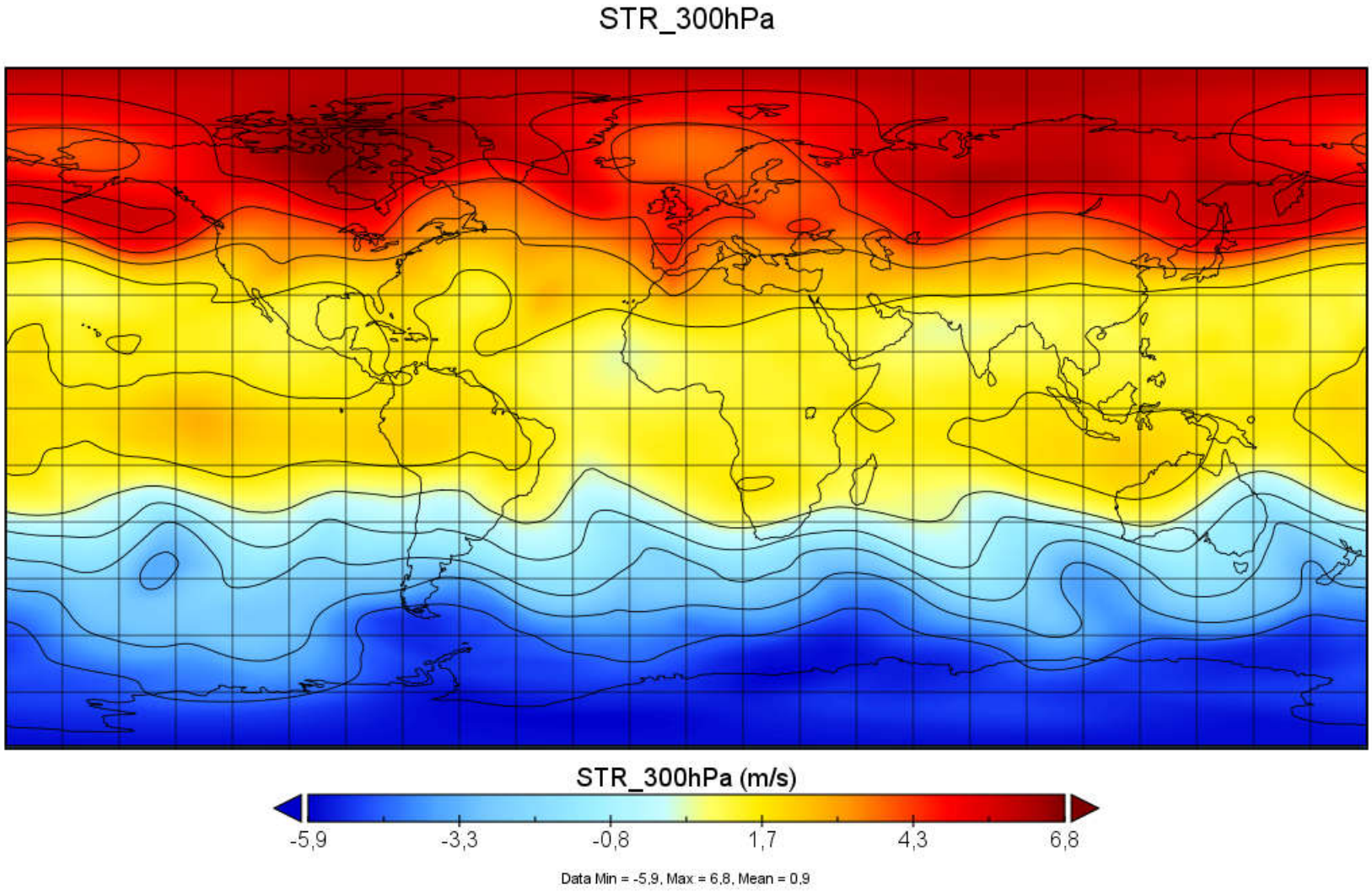
$$\delta = \nabla \cdot \boldsymbol{v}$$

divg_300hPa



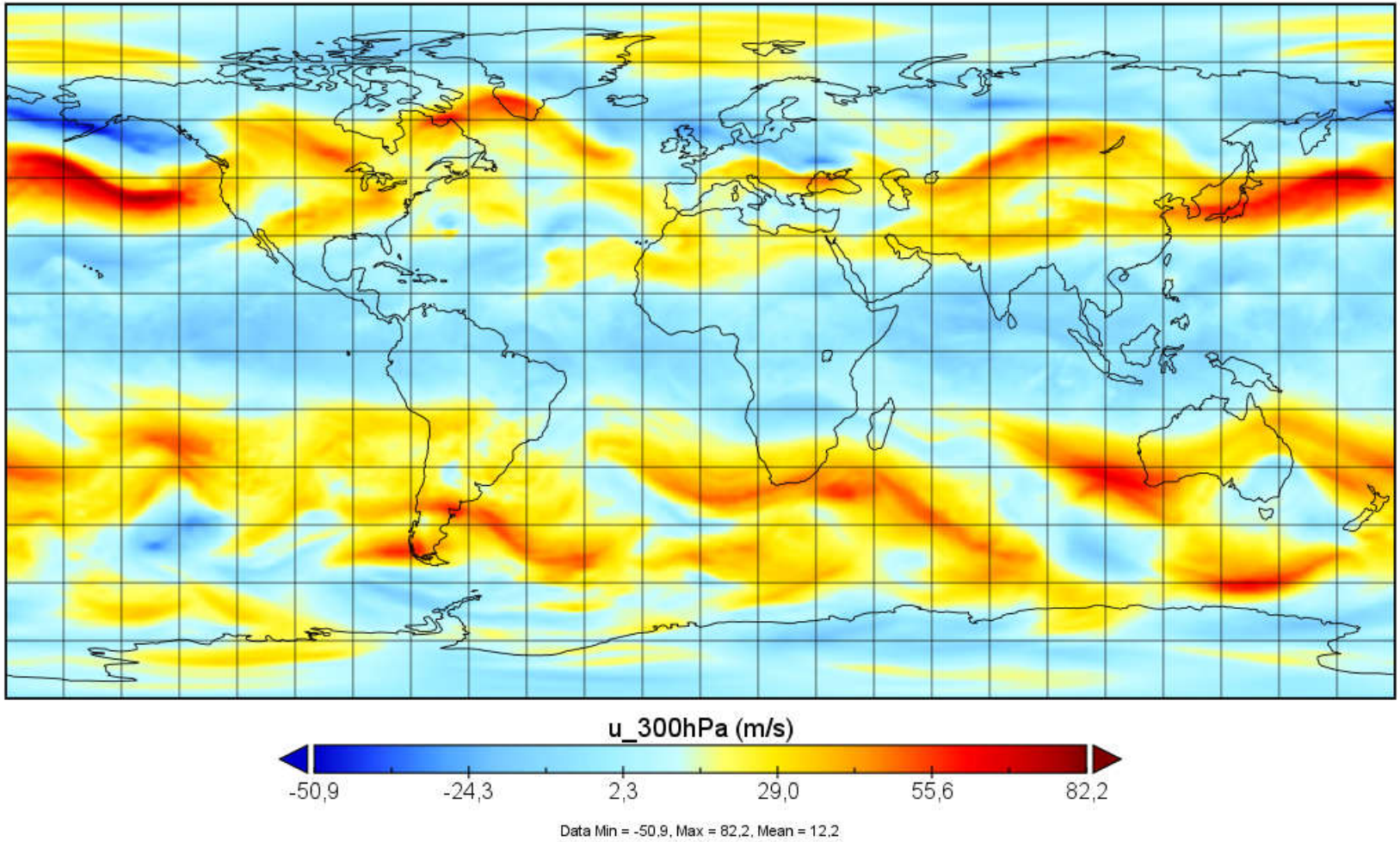
$$\zeta = \boldsymbol{k}\cdot(\nabla \times \boldsymbol{v}) = \frac{1}{R\cos\theta}\left[\frac{\partial v}{\partial \lambda} - \frac{\partial u \cos\varphi}{\partial \varphi}\right] = \nabla^2 \psi$$

$$\mathrm{STR} = \psi/(\pi R)$$



Solution of Helmholtz equation after spherical harmonics decomposition (Swarztrauber et al. 1993) using SPHEREPACK (2011)

u_300hPa

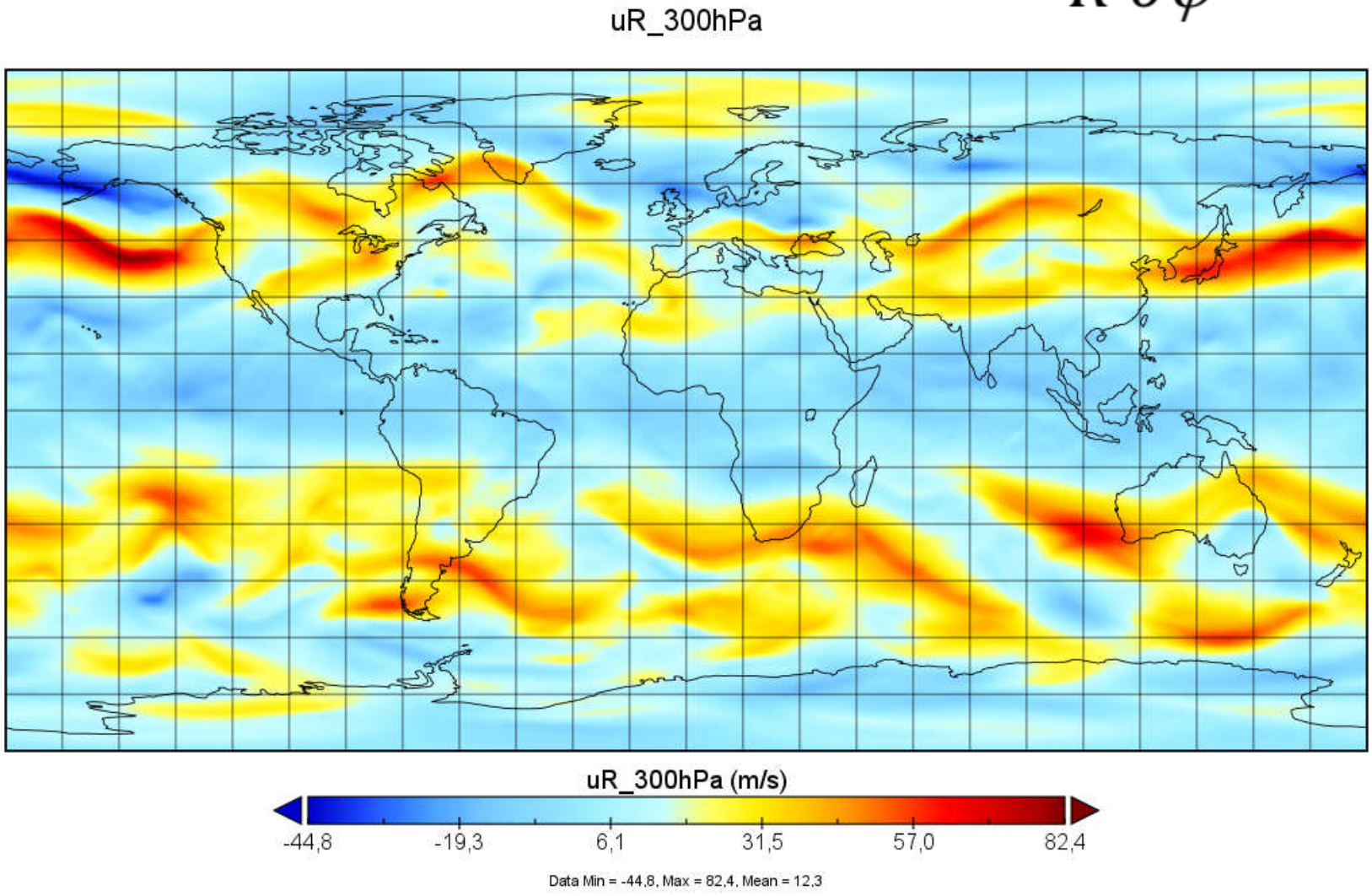


u_300hPa (m/s)

-50,9 -24,3 2,3 29,0 55,6 82,2

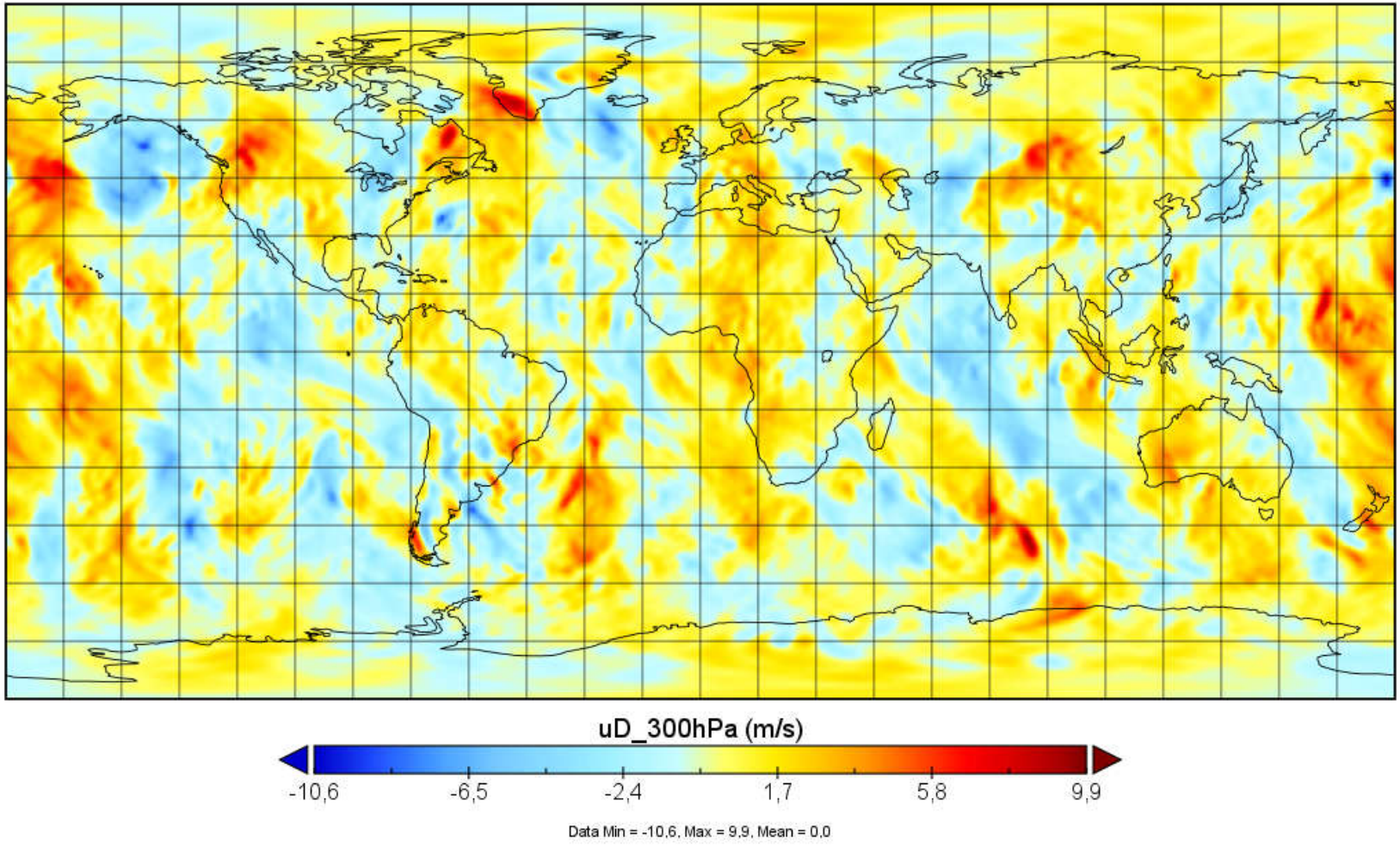
Data Min = -50,9, Max = 82,2, Mean = 12,2

$$u_R = -\frac{1}{R}\frac{\partial \psi}{\partial \varphi},$$

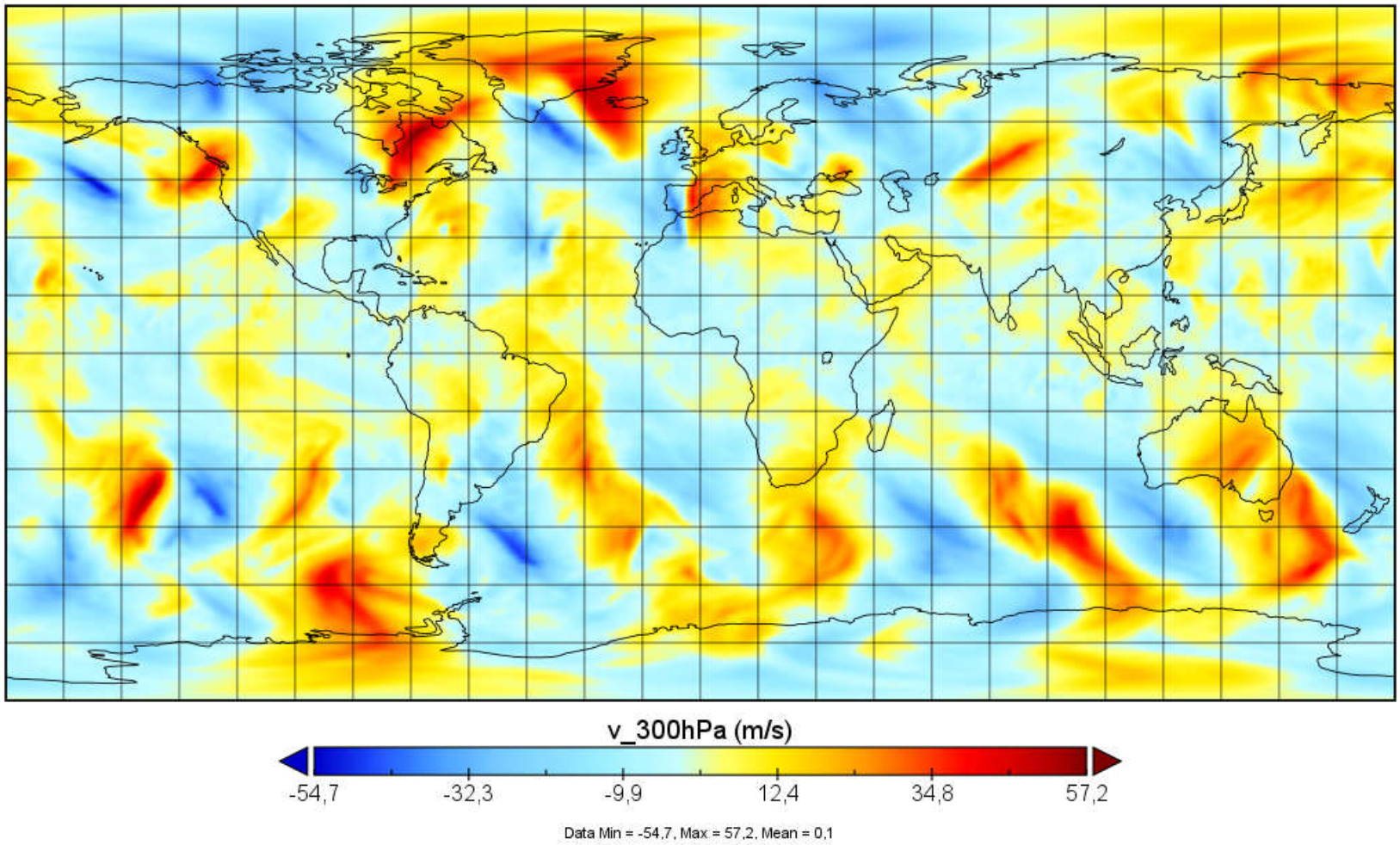


$$u_D = \frac{1}{R\cos\varphi}\frac{\partial\chi}{\partial\lambda}$$

uD_300hPa

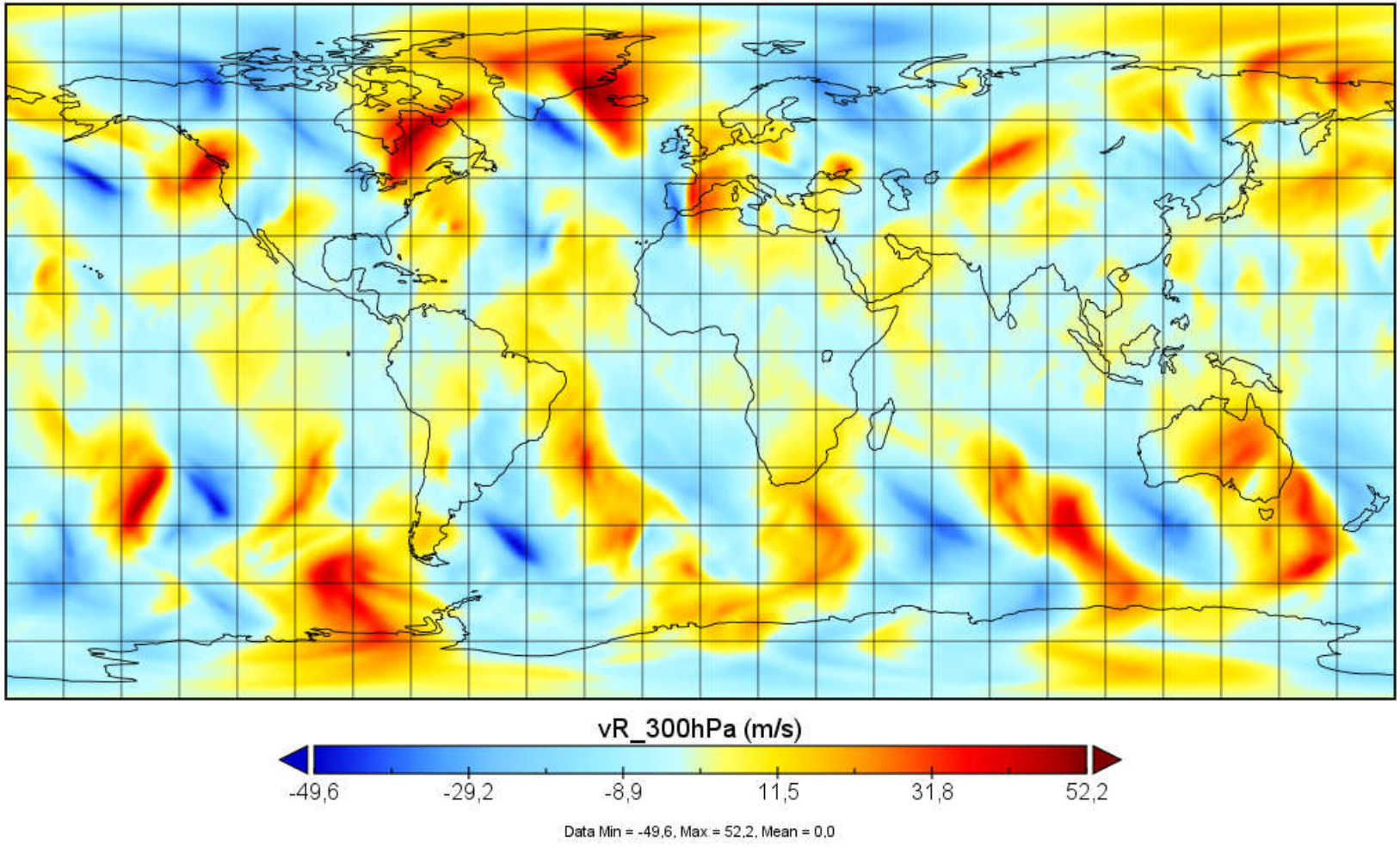


v_300hPa

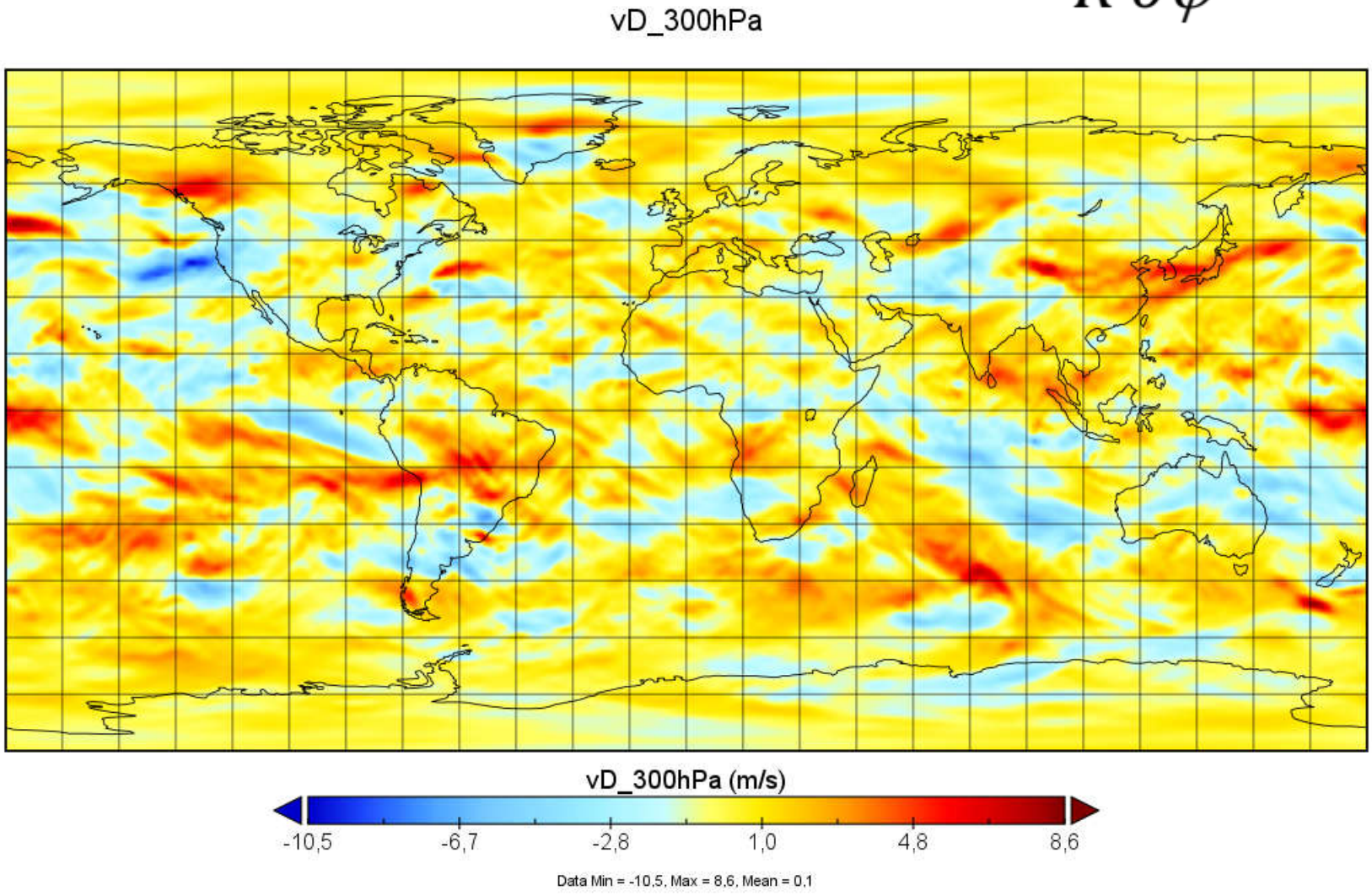


$$v_R = \frac{1}{R \cos \varphi} \frac{\partial \psi}{\partial \lambda}$$

vR_300hPa

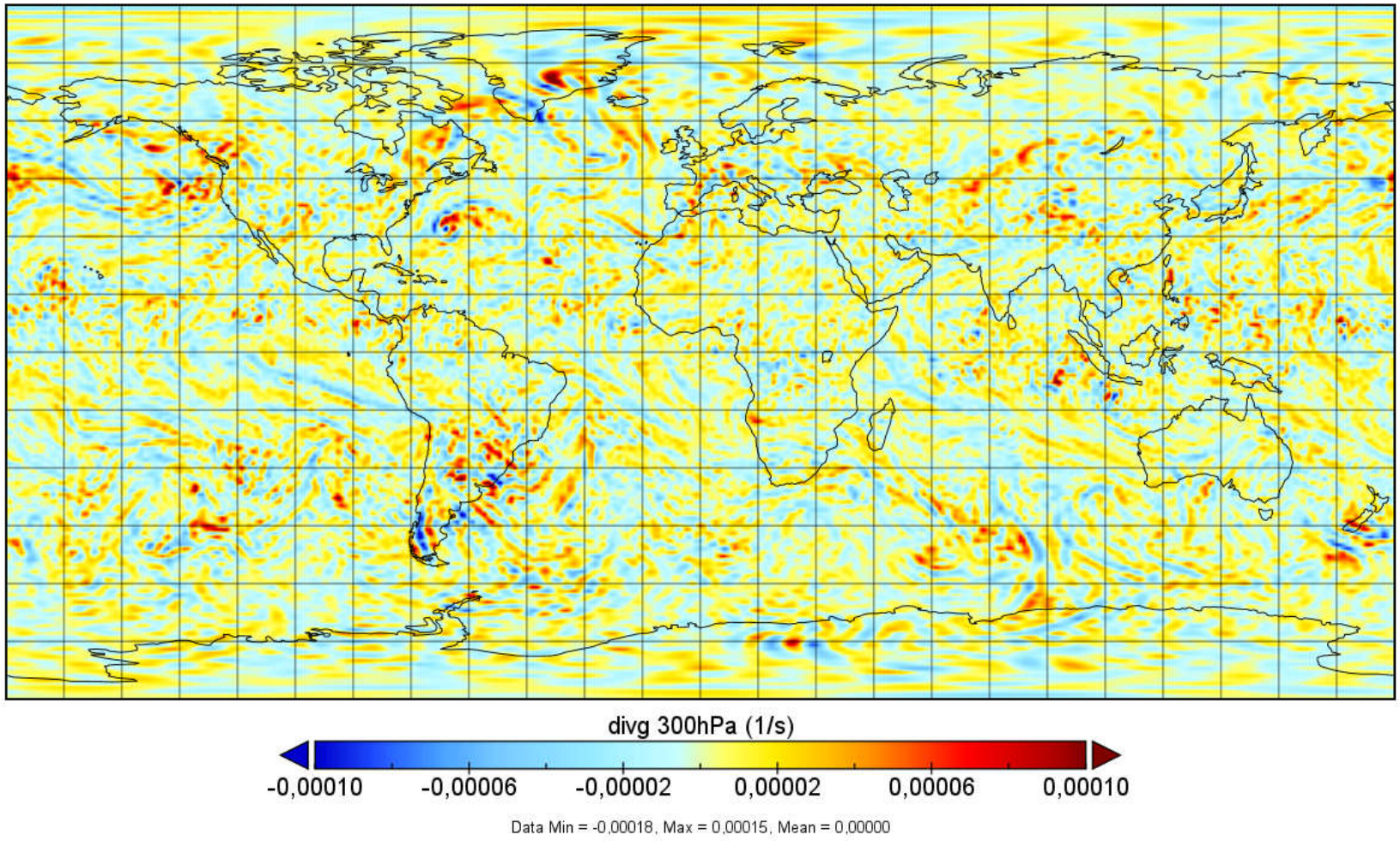


$$v_D = \frac{1}{R}\frac{\partial \chi}{\partial \varphi}$$



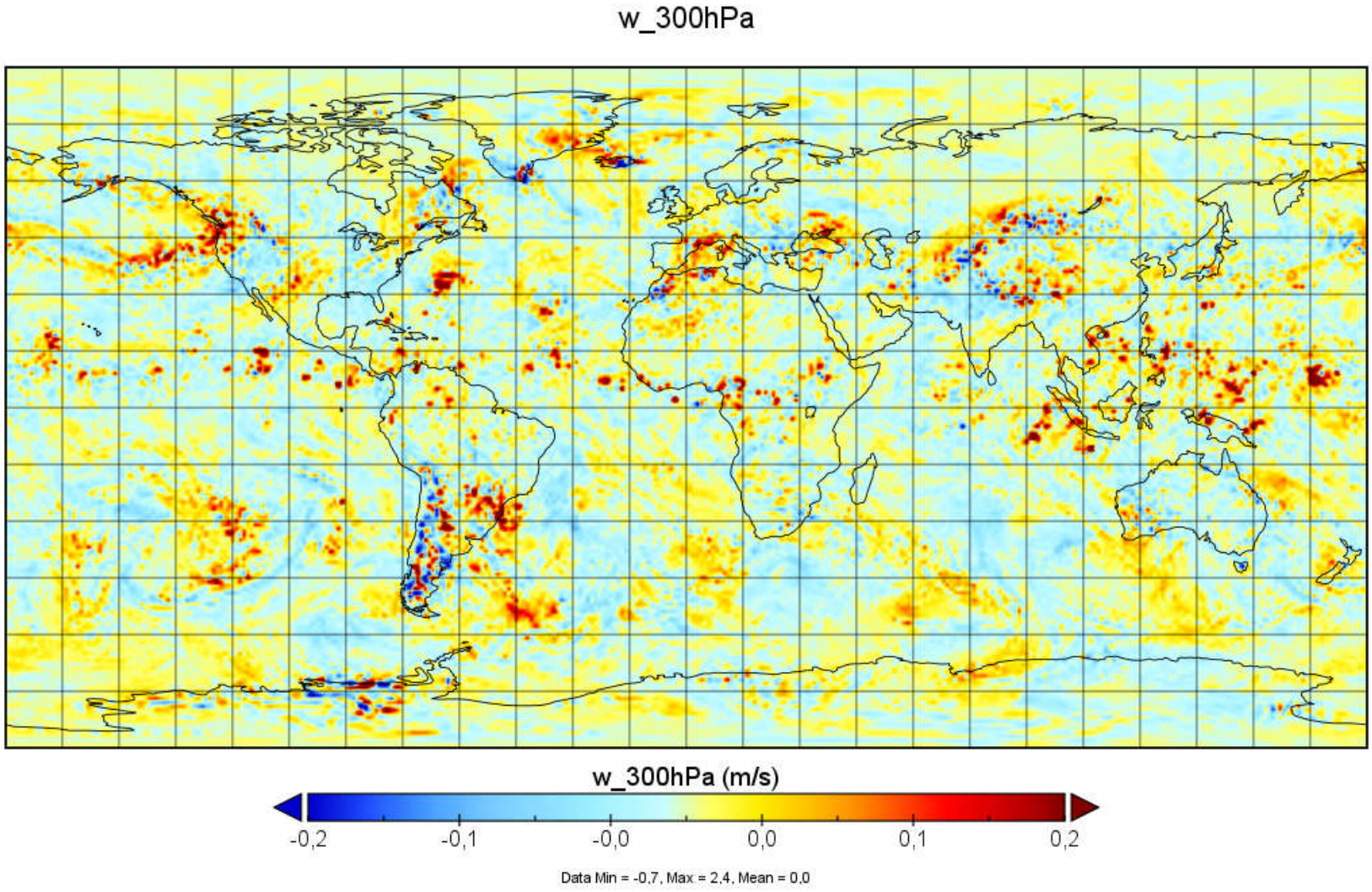
$$\delta = \nabla \cdot \boldsymbol{v} = \frac{1}{R\cos\varphi}\frac{\partial u}{\partial \lambda} + \frac{1}{R\cos\varphi}\frac{\partial (v\cos\varphi)}{\partial \varphi} = \nabla^2 \chi$$

divg 300hPa



$$\frac{\partial w}{\partial z} = -\left(\frac{\partial u_D}{\partial x} + \frac{\partial v_D}{\partial y}\right) = -\delta, \quad w(z) - w(0) = -\int_0^z \delta \, dz$$

Note: only the divergent velocity components determine vertical wind



Exchange between potential and horizontal kinetic energy Catalyzed by divergent horizontal velocity (w)

$$\boldsymbol{v} = \boldsymbol{k} \times \nabla\psi + \nabla\chi$$

work of divergent motions against gradients of geopotential

potential + internal energy

rotational energy

divergent energy

$$\frac{\partial}{\partial t}(\overline{\overline{P}} + \overline{\overline{I}}) = + \overline{\overline{\nabla\chi\,\nabla\Phi}} + \overline{\overline{dQ}}$$

$$\frac{\partial}{\partial t}\overline{\overline{\frac{(\nabla\psi)^2}{2}}} = - \overline{\overline{(f + \nabla^2\psi)\,\nabla\chi\,\nabla\psi}} - \overline{\overline{\nabla^2\chi\,\frac{(\nabla\psi)^2}{2}}} - \overline{\overline{\omega\,\boldsymbol{k}\left[\nabla\psi \times \nabla\frac{\partial\chi}{\partial\psi}\right]}}$$

$$\frac{\partial}{\partial t}\overline{\overline{\frac{(\nabla\chi)^2}{2}}} = - \overline{\overline{\nabla\chi\,\nabla\Phi}} + \overline{\overline{(f + \nabla^2\psi)\,\nabla\chi\,\nabla\psi}} + \overline{\overline{\nabla^2\chi\,\frac{(\nabla\psi)^2}{2}}} + \overline{\overline{\omega\,\boldsymbol{k}\left[\nabla\psi \times \nabla\frac{\partial\chi}{\partial p}\right]}}$$

(Wippermann, 1957;
see also Lorenz 1960; Wiin-Nielsen, 1968; Chen and Wiin-Nielsen, 1976)

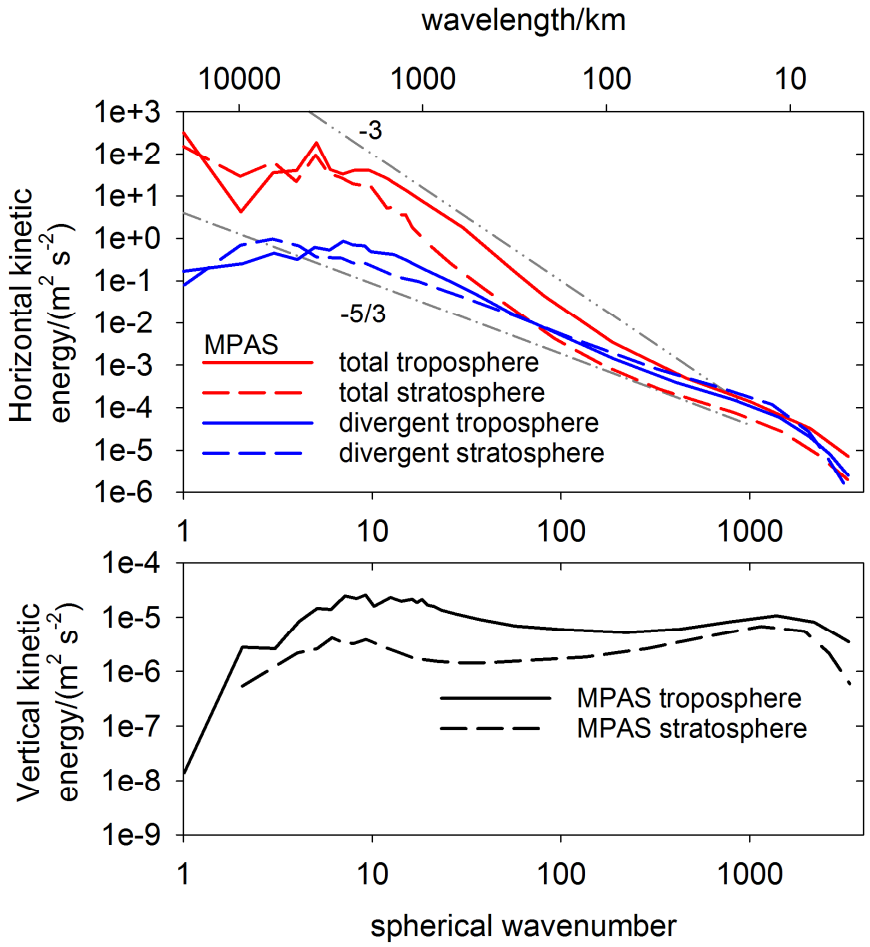
What do we know about horizontal w spectra

from simulations

from aircraft measurements?

Model for Prediction Across Scales: MPAS, 3 km resolution

Global numerical simulation spectra



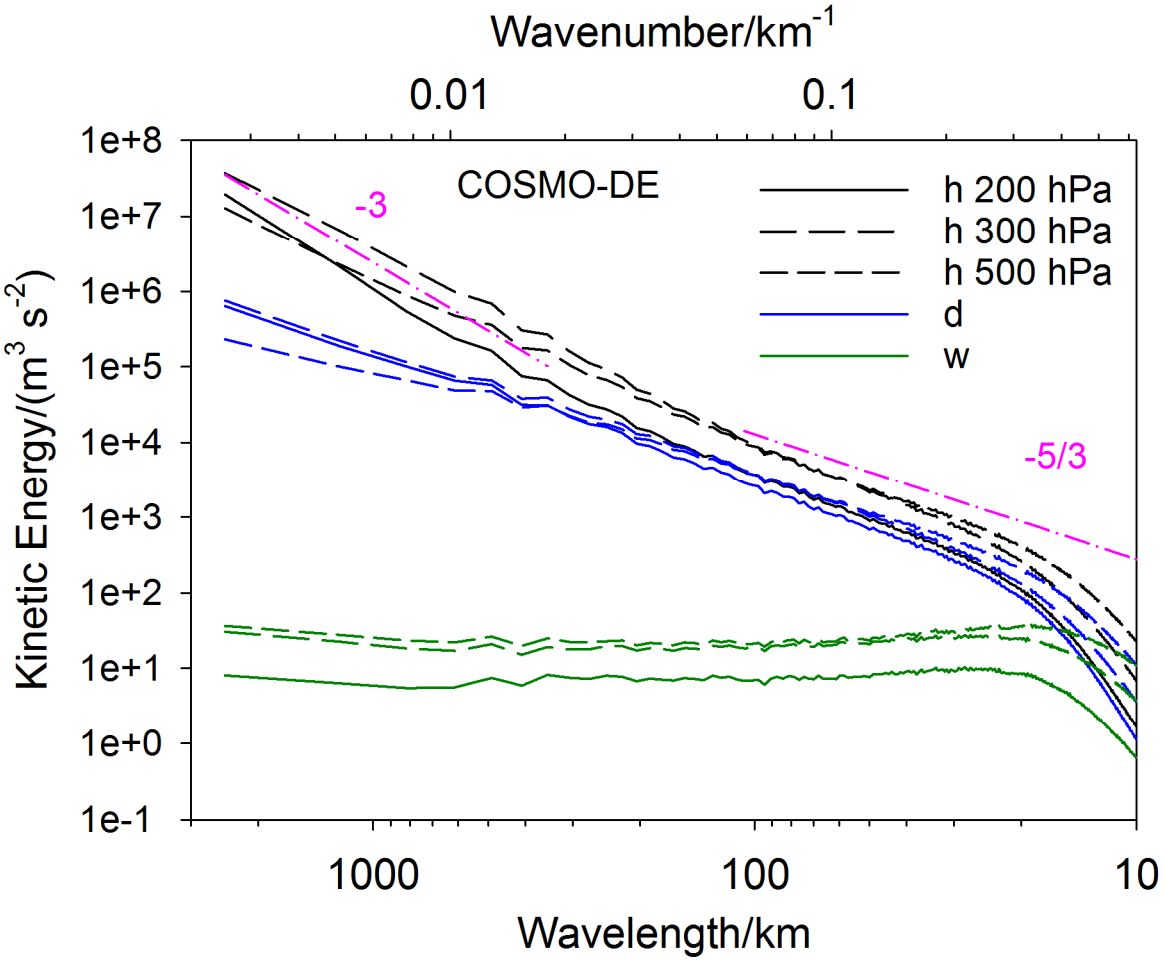
-3 and -5/3-spectrum as in canonical spectrum

near -2 and -5/3 divergent spectrum, $d = E_d/(E_d+E_r) \approx 0.5$ at mesoscales

different w-spectra in TRO and STR with unexplained maxima

(Skamarock et al., 2014)

COSMO-DE: 3 years of regional numerical weather prediction data



Data courtesy Tobias Selz

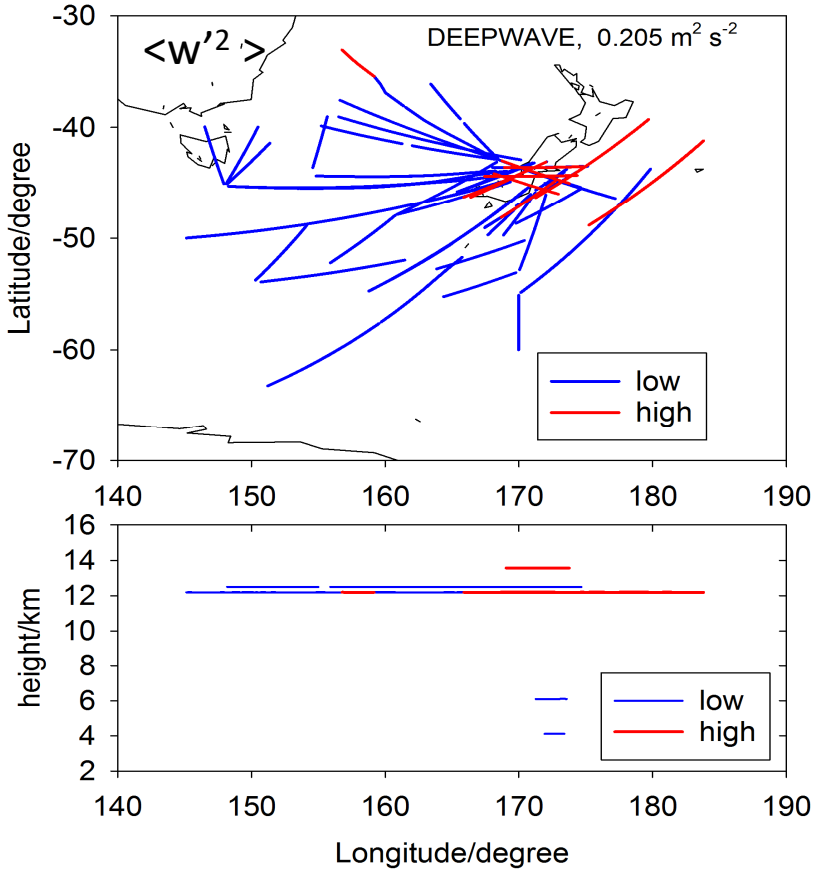
spectrum only slightly different from canonical spectrum

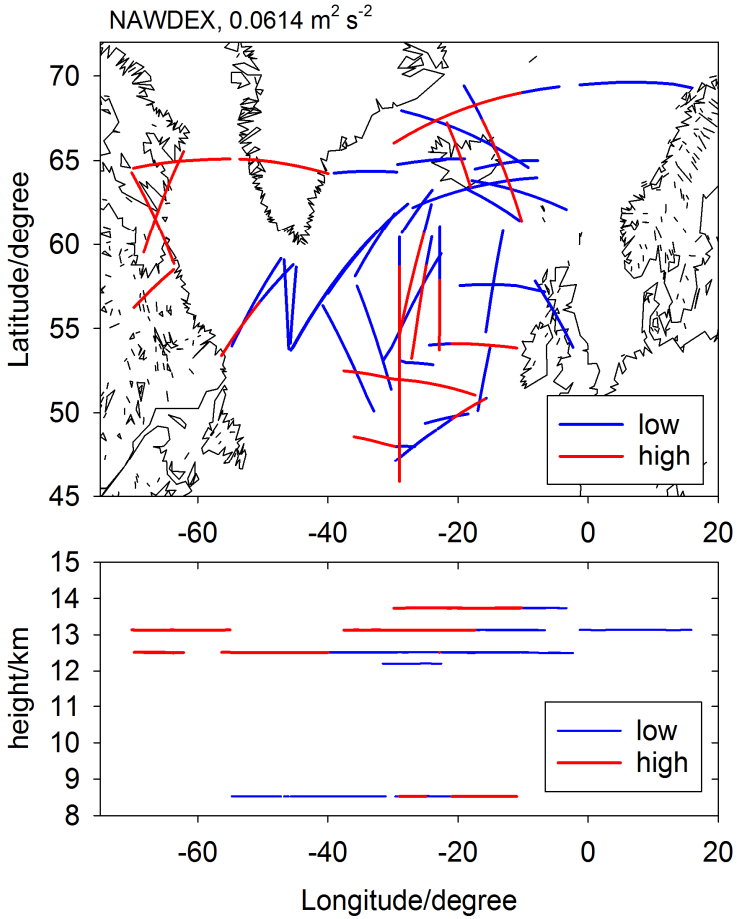
-2 and -5/3 divergent spectrum, $d \approx 0.5$ at mesoscales

flat w-spectrum with maximum possibly due to convection

(Selz, Bierdel, Craig, JAS, 2019)

DEEPWAVE (D, 2014) and NAWDEX (N, 2016) airborne measurements in the upper troposphere/lower stratosphere (UTLS), 180 and 107 legs, 2048 s each

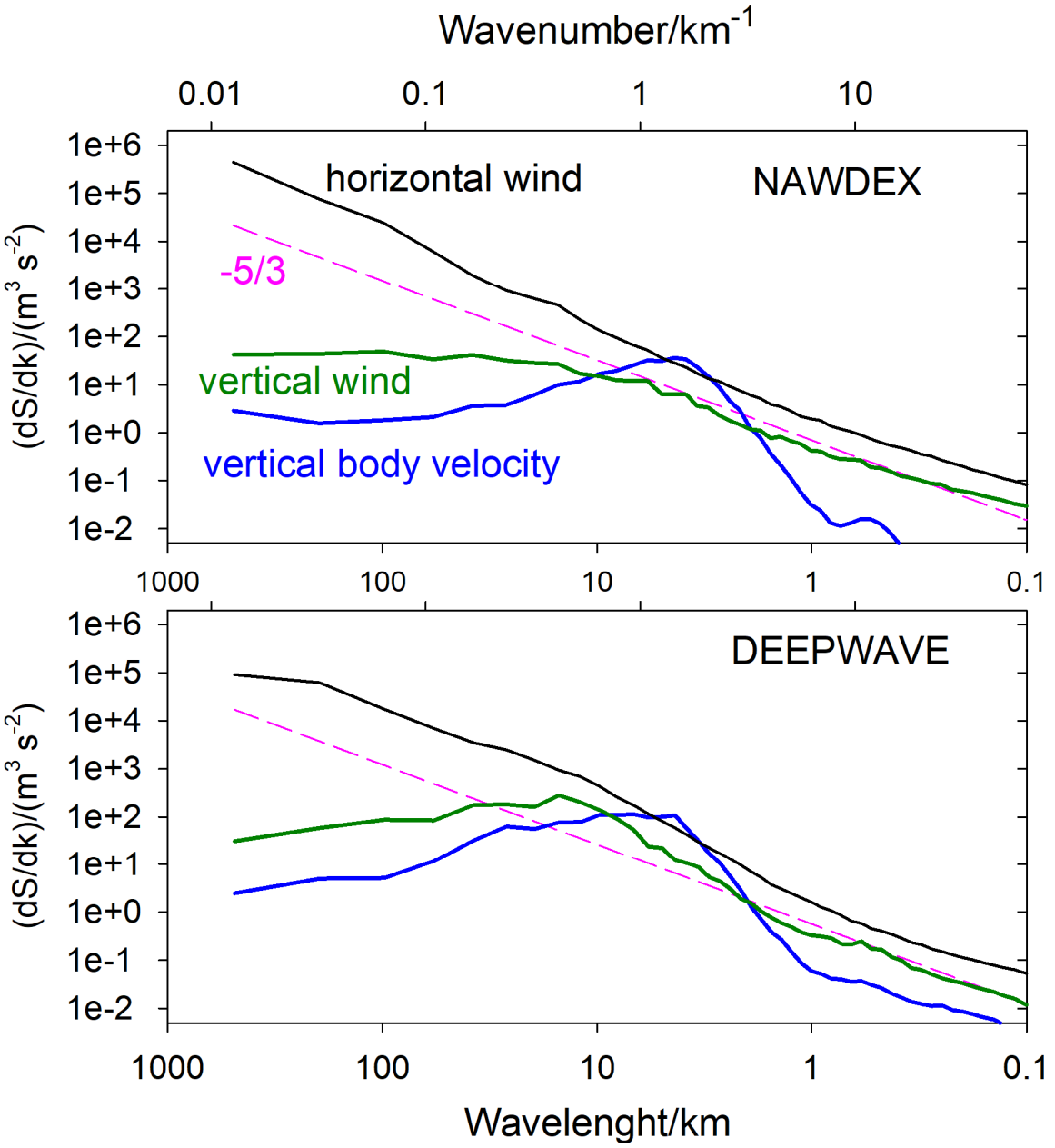




Fritts et al. (2016),
NSF-GV (HIAPER) NCAR EOL

Schäfler et al. (2018),
HALO DLR-FX

Measured velocity spectra, mean over all legs



How to explain the w spectra?

Impact of vertical body motions on w measurement?

Hypothesis:

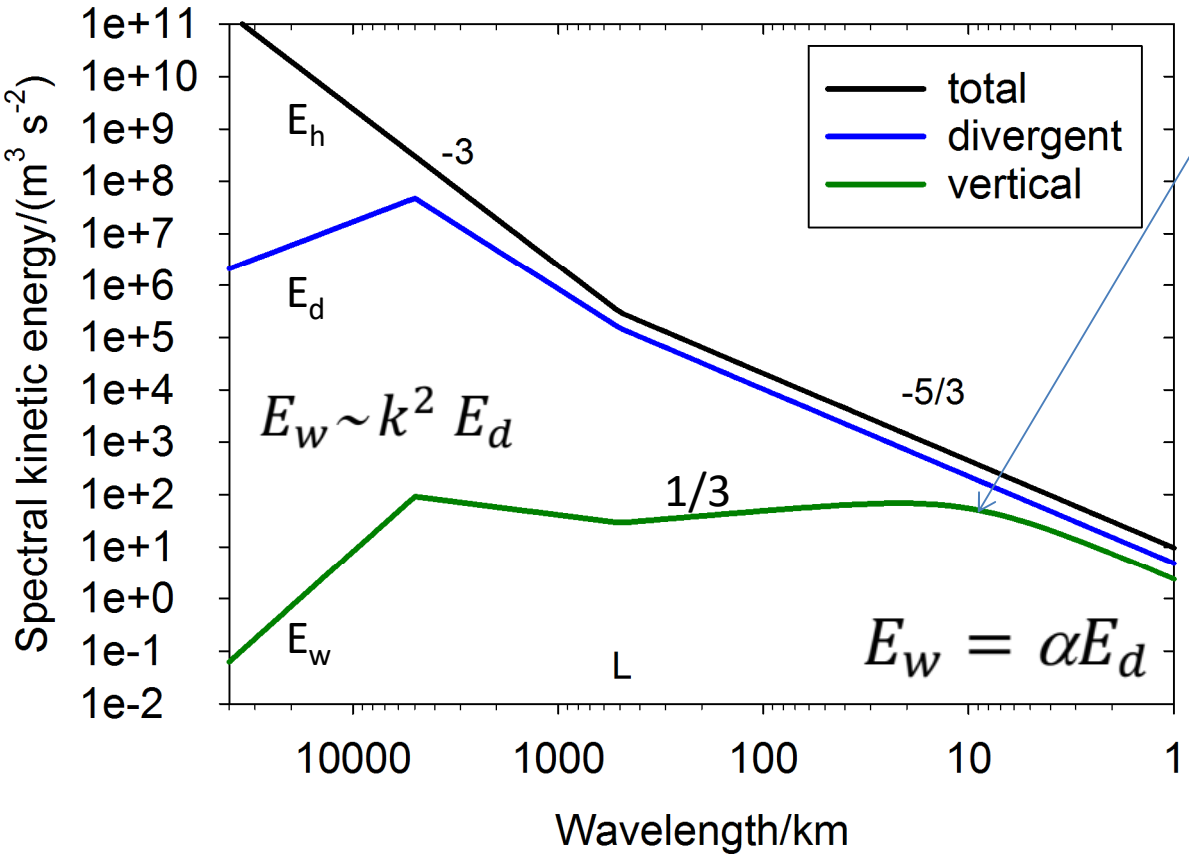
The spectrum of vertical velocities E_w,
as a function of wavenumber k and height h,
is related to the spectrum of horizontal velocities E_h
1) at large scales: to E_d by continuity,
2) at small scales: to E_h by dynamics towards local isotropy

$$E_w(k,h) = \frac{\alpha(h_{\text{eff}}\,k)^2}{\alpha+(h_{\text{eff}}\,k)^2}\,d\;E_h(k,h),\; h_{\text{eff}} = \beta\,h$$

$\alpha \approx 0.5$ for stratified turbulence
$\beta \approx 0.1$ to 0.5, depending on vertical variability of horizontal motions
$d \approx 0.5$, divergent energy fraction at mesoscales

For formal derivation, see J. Atmos Sci (2019)

A schematic sketch of E_w



A mesoscale maximum in the w-spectrum occurs if the divergent horizontal velocity spectrum E_d has a slope flatter than -2

$$E_w(k,h) = \frac{\alpha(h_{\text{eff}}\,k)^2}{\alpha+(h_{\text{eff}}\,k)^2} E_d(k,h),\ h_{\text{eff}} = \beta\, h$$

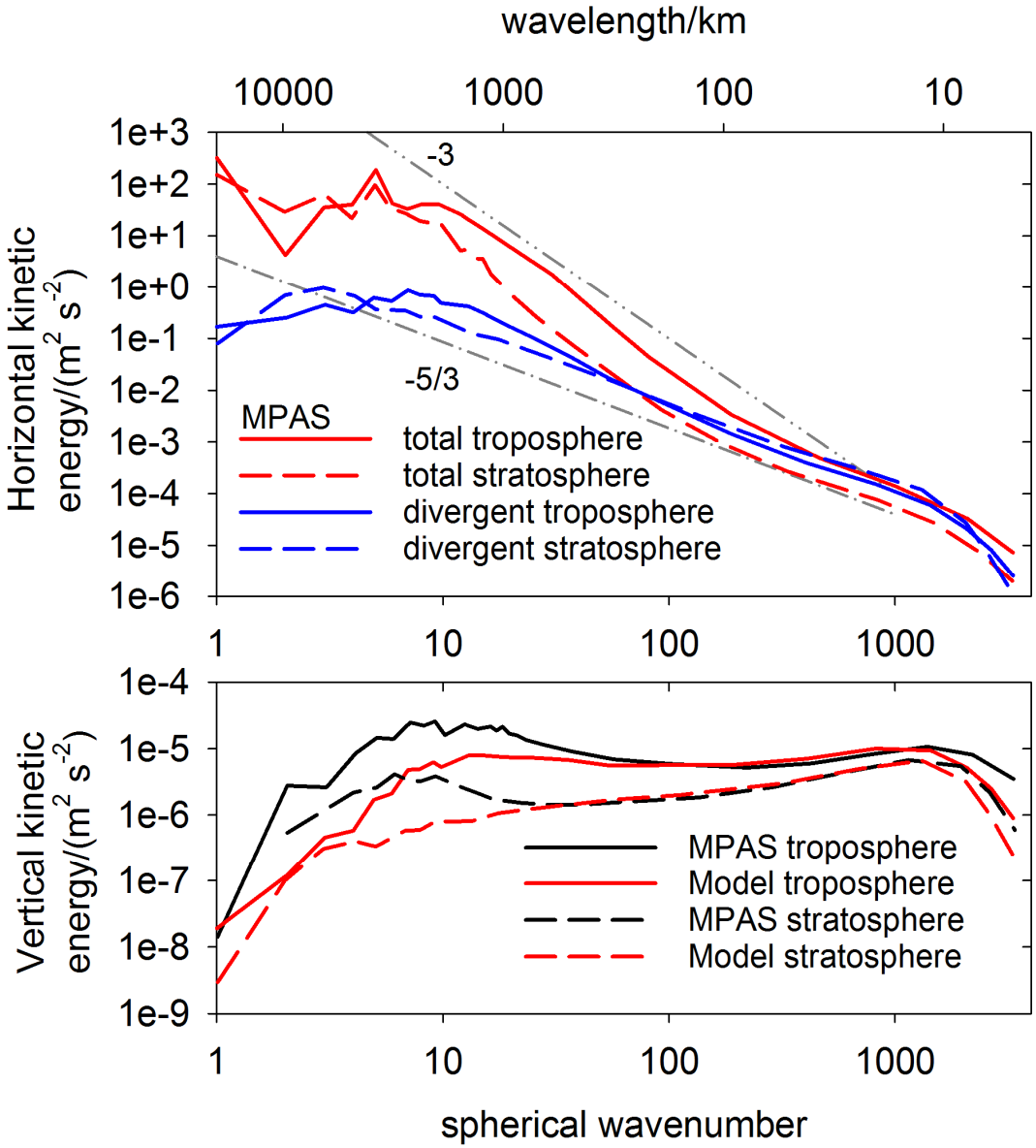
h=10 km, α =1/2, β =0.11, d=1/2

Comparison of the model spectrum with

global simulations

aircraft measurements

Comparison of the w-model to the global simulation w-spectra

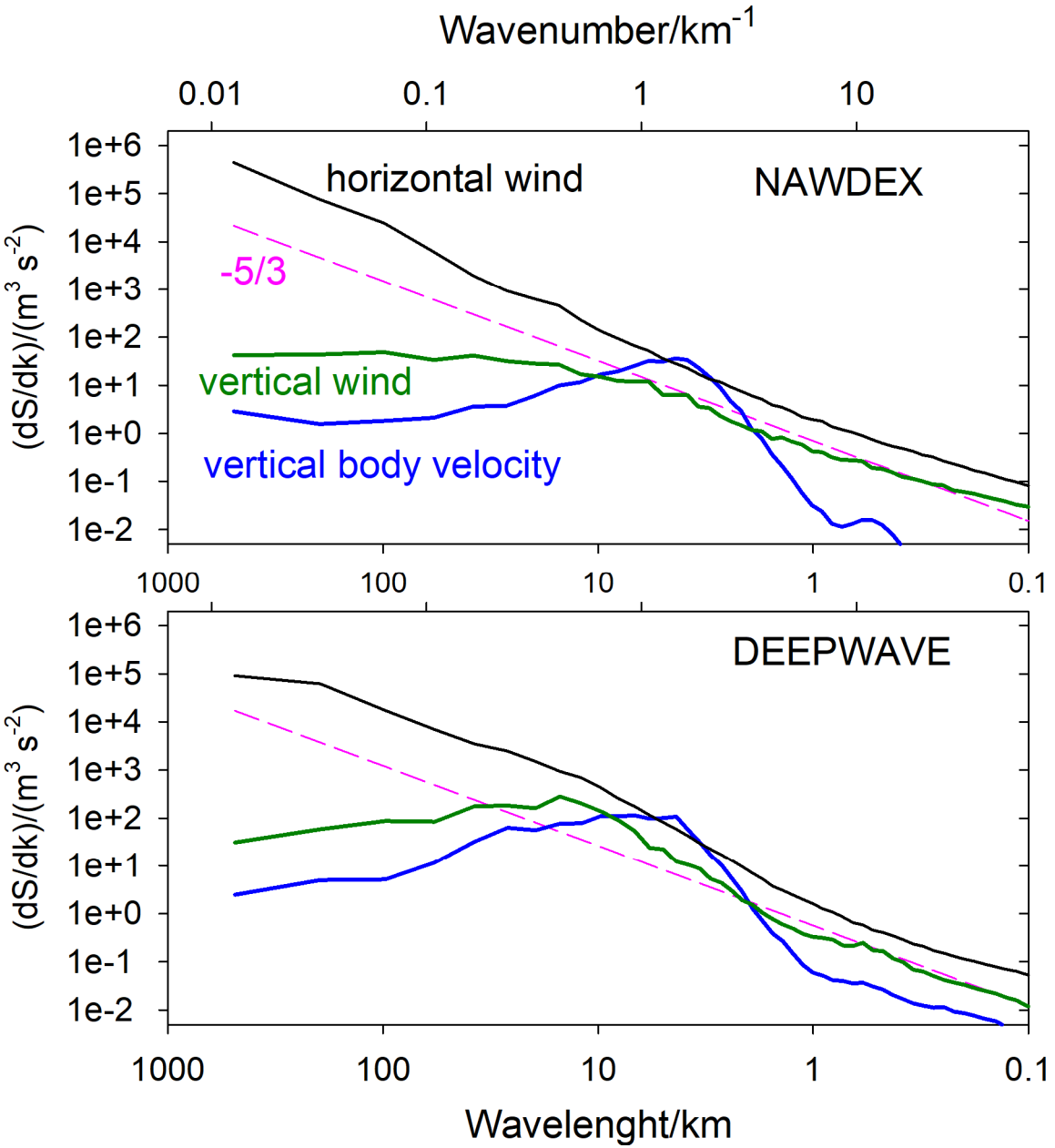


The proposed model is consistent with the MPAS results.

Some deviations were to be expected because of

- non-2d-isotropy
- surface orography

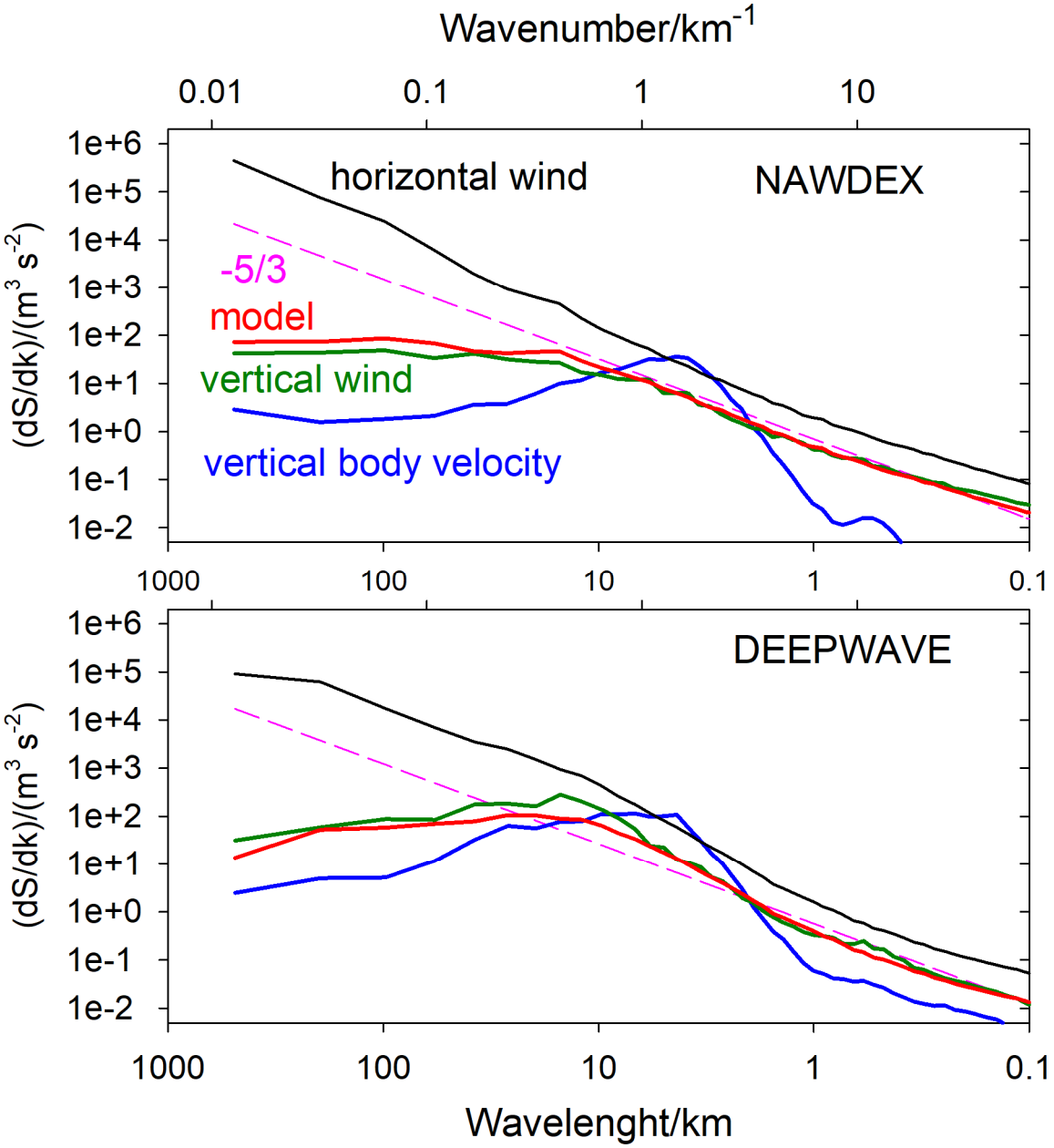
Again: Mean measured velocity spectra



How to explain the w spectra?

Impact of vertical body motions?

Mean measured and modelled velocity spectra

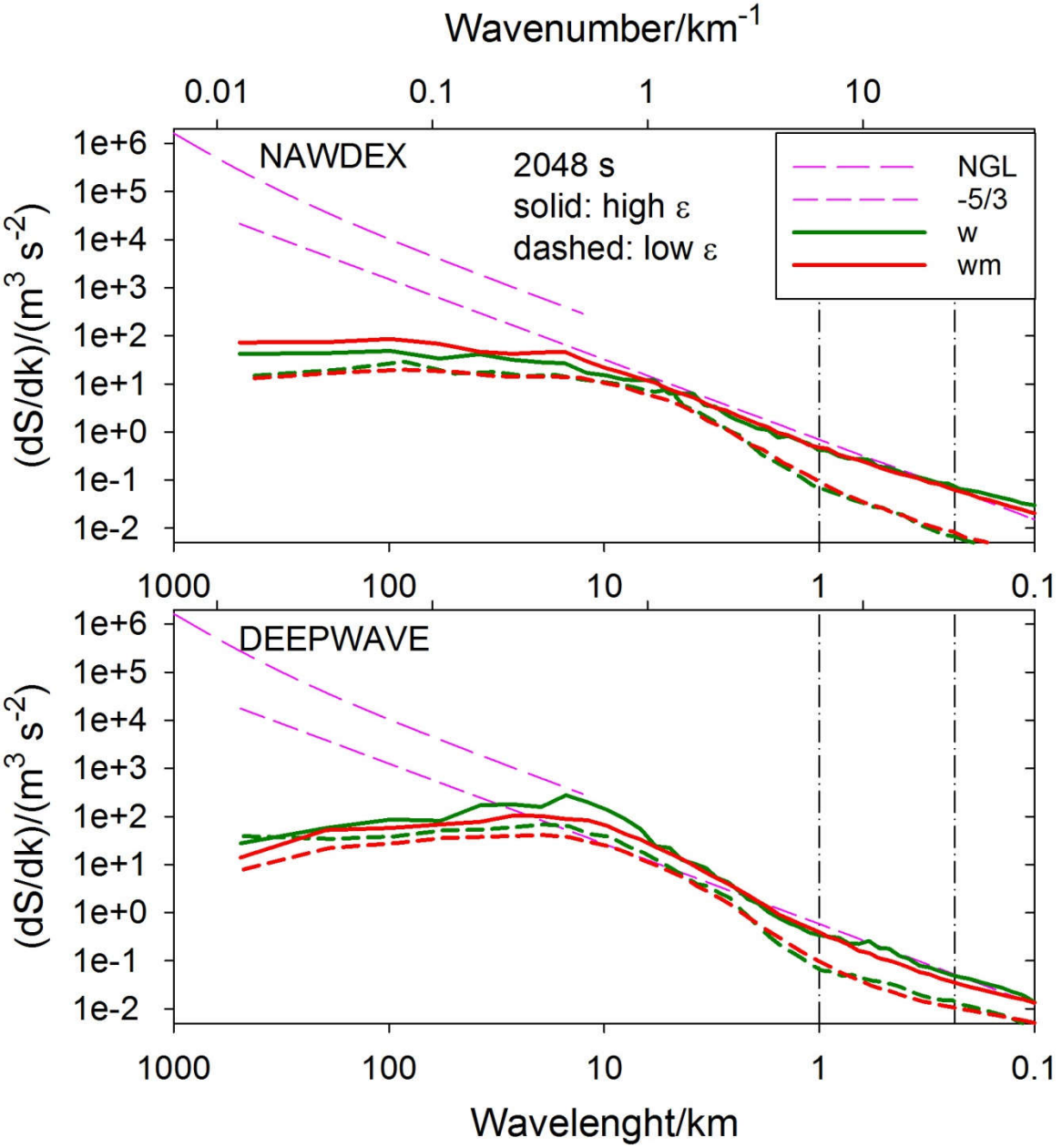


How to explain the spectra?

Impact of vertical body motions?

Model for w-spectrum

Comparison of measured and modeled w-spectra

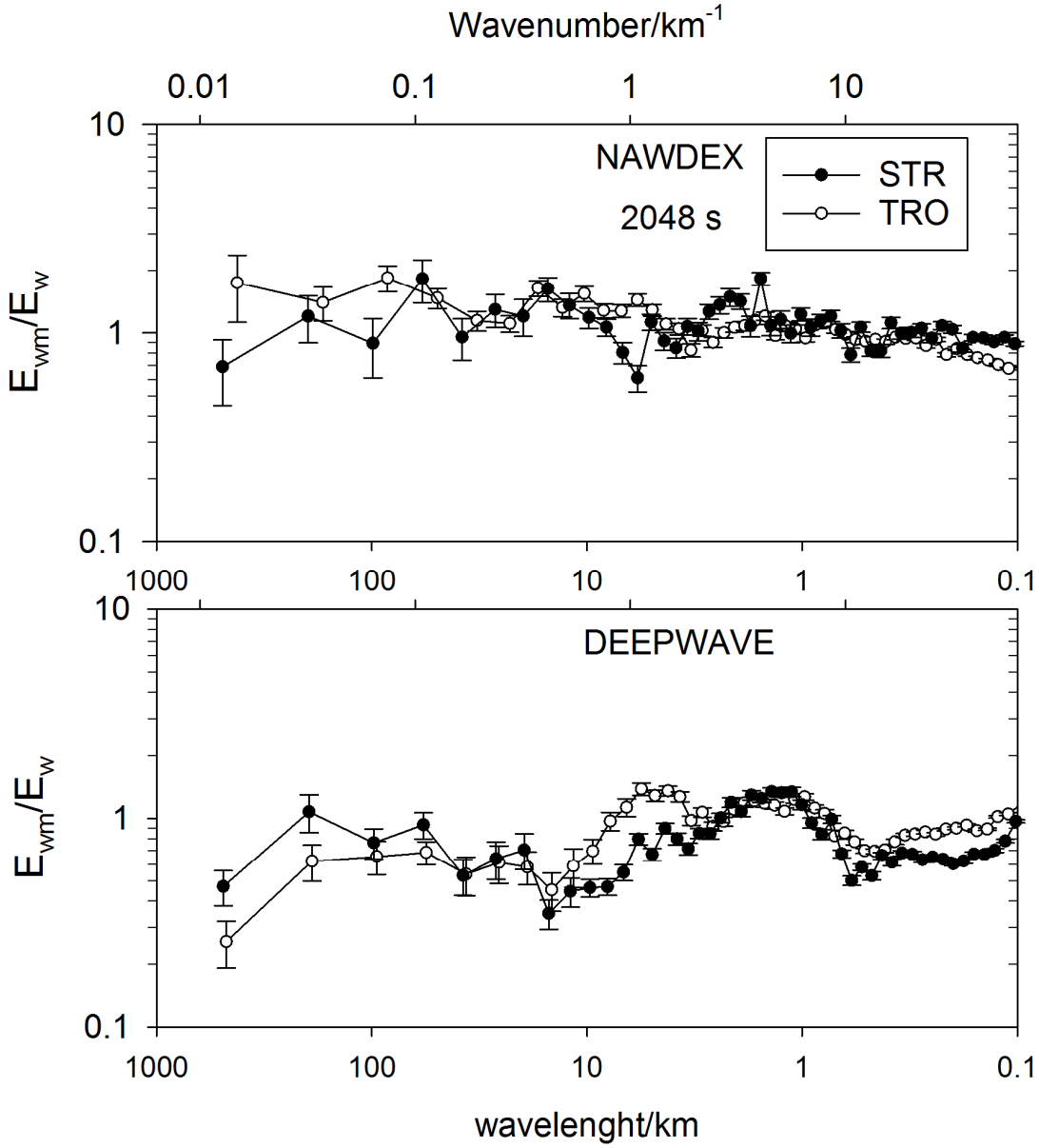


Model
and
observations

similar
at high and low ε
in both experiments

h as measured
d=1/2, β=0.11, α=1/2

Ratio of measured and modeled w-spectra

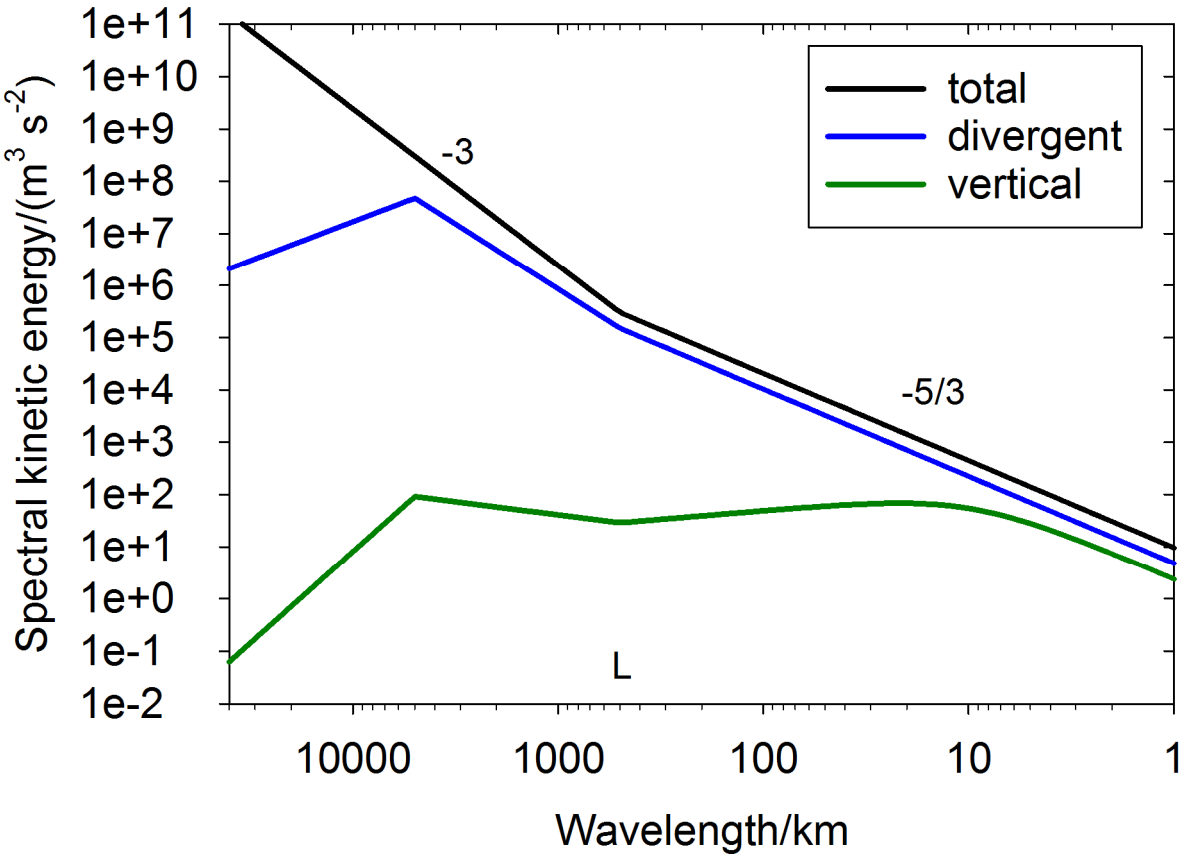


about 30 % deviations

Implications

- Transition scale L increases for stronger w-variance
- Potential energy follows w spectrum at small scales
- The scales carrying w-variance are of order height

Schematic sketch of E_w for two transition scales L

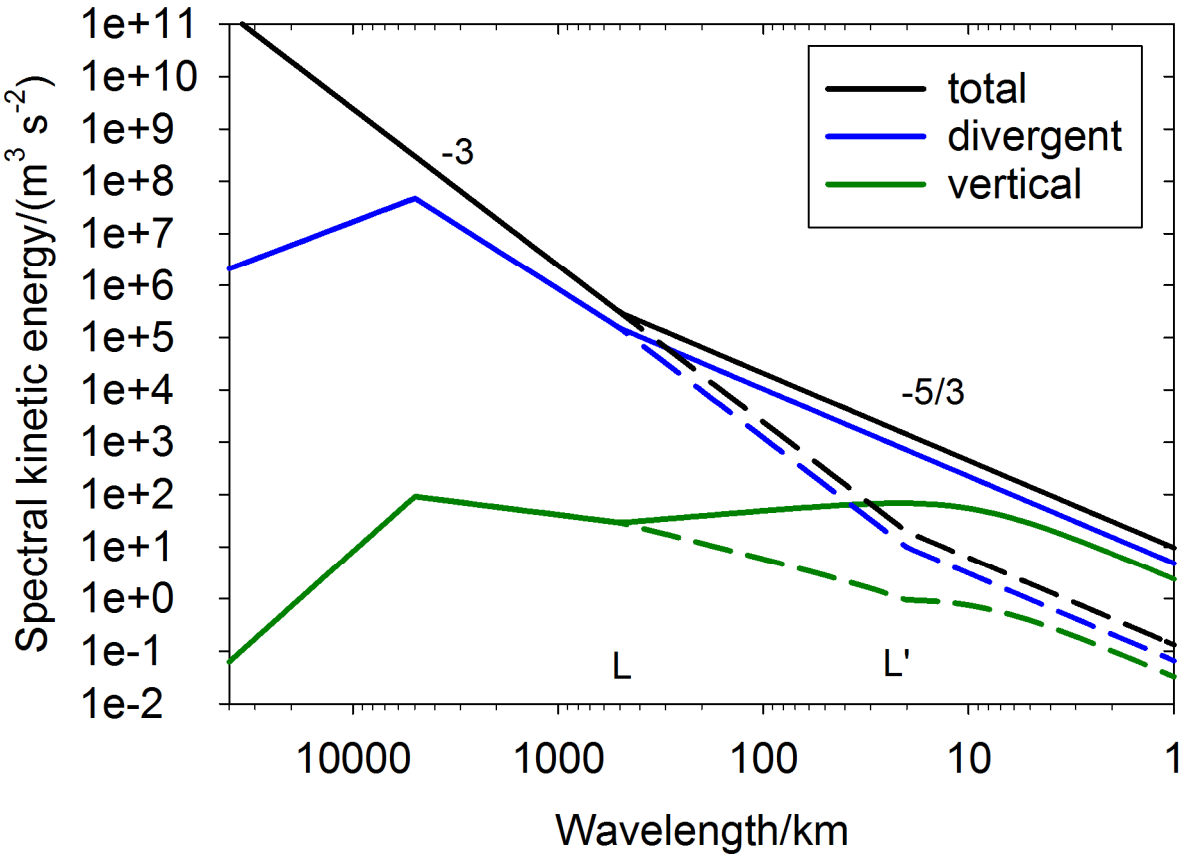


A shift in the transition scale L has strong impact on w-spectra.

This suggests dynamical importance of vertical motions for this slope transition

$$E_w(k,h) = \frac{\alpha(h_{\text{eff}}\,k)^2}{\alpha+(h_{\text{eff}}\,k)^2} E_d(k,h),\ h_{\text{eff}} = \beta\, h$$

Schematic sketch of E_w for two transition scales L

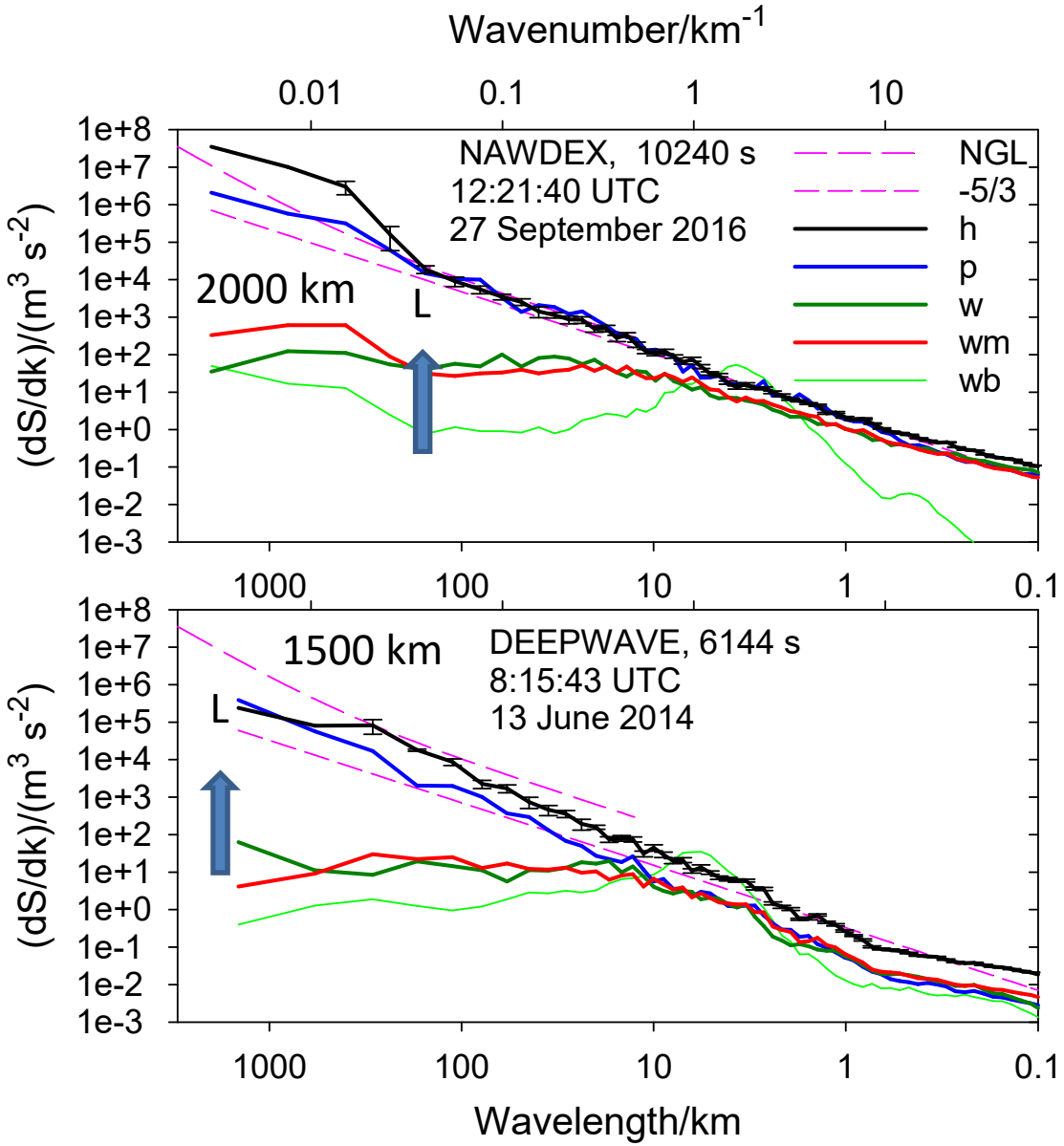


A shift in the transition scale L has strong impact on w-spectra.

This suggests dynamical importance of vertical motions for this slope transition

$$E_w(k,h) = \frac{\alpha(h_{\text{eff}}\,k)^2}{\alpha+(h_{\text{eff}}\,k)^2} E_d(k,h),\ h_{\text{eff}} = \beta\, h$$

Measured and modeled w-spectra for the longest legs



<w'2> =

0.024 $m^2 s^{-2}$
L= 200 km

0.068 $m^2 s^{-2}$ L
L>1000 km

Temperature spectra are similar to wind spectra

Spectrum of potential energy

$$E_p(k) = \frac{1}{2}\frac{g^2}{N^2 T_0^2} E_T(k)$$

$$\frac{1}{2}\frac{g^2}{N^2 T_0^2} \approx (2 \text{ to } 10)\,\frac{\mathrm{m}^2}{\mathrm{s}^2\mathrm{K}^2}$$

$$E_p(k) \approx E_u(k) \approx E_v(k)$$

w converts kinetic into potential energy and backward

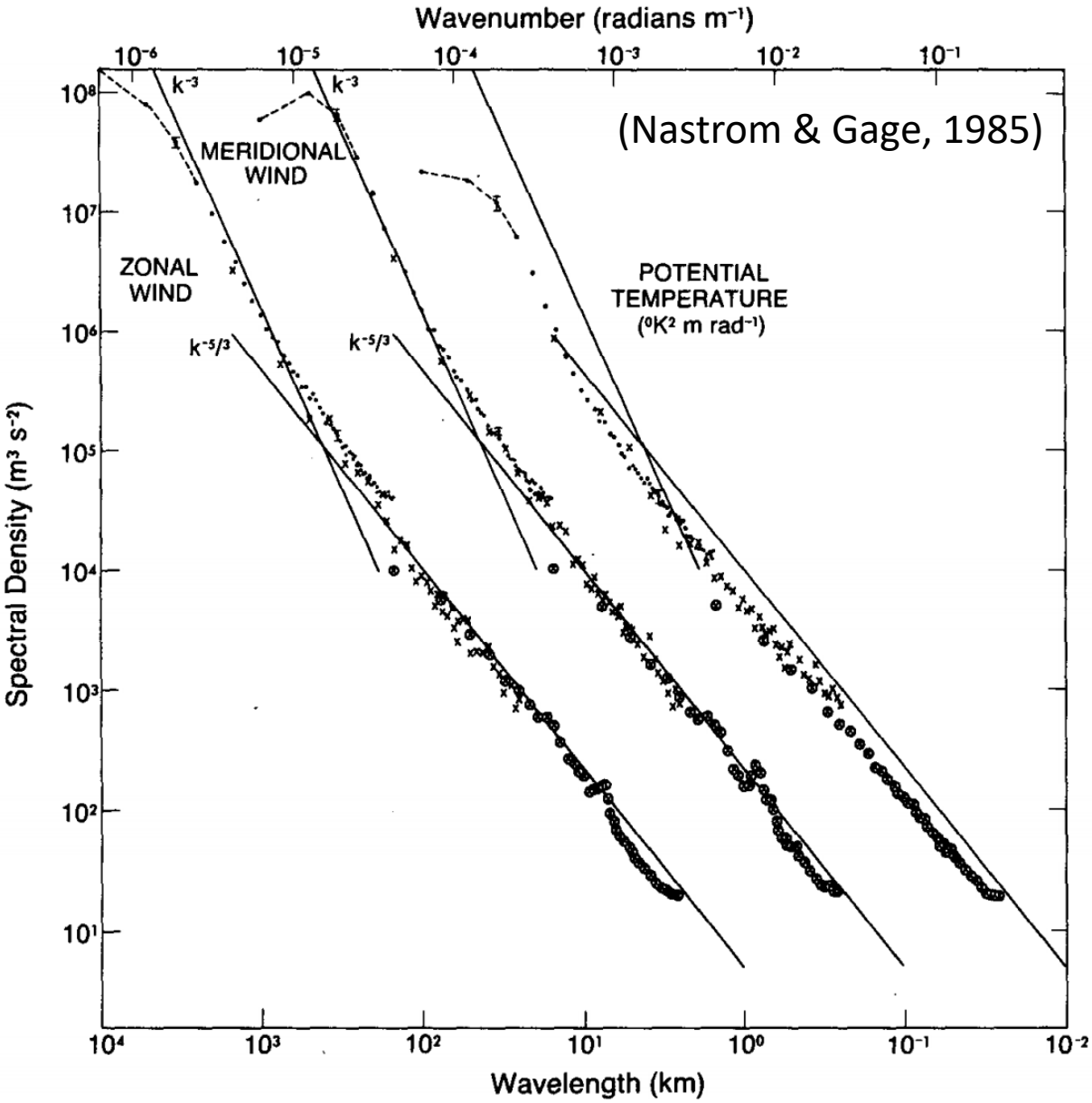
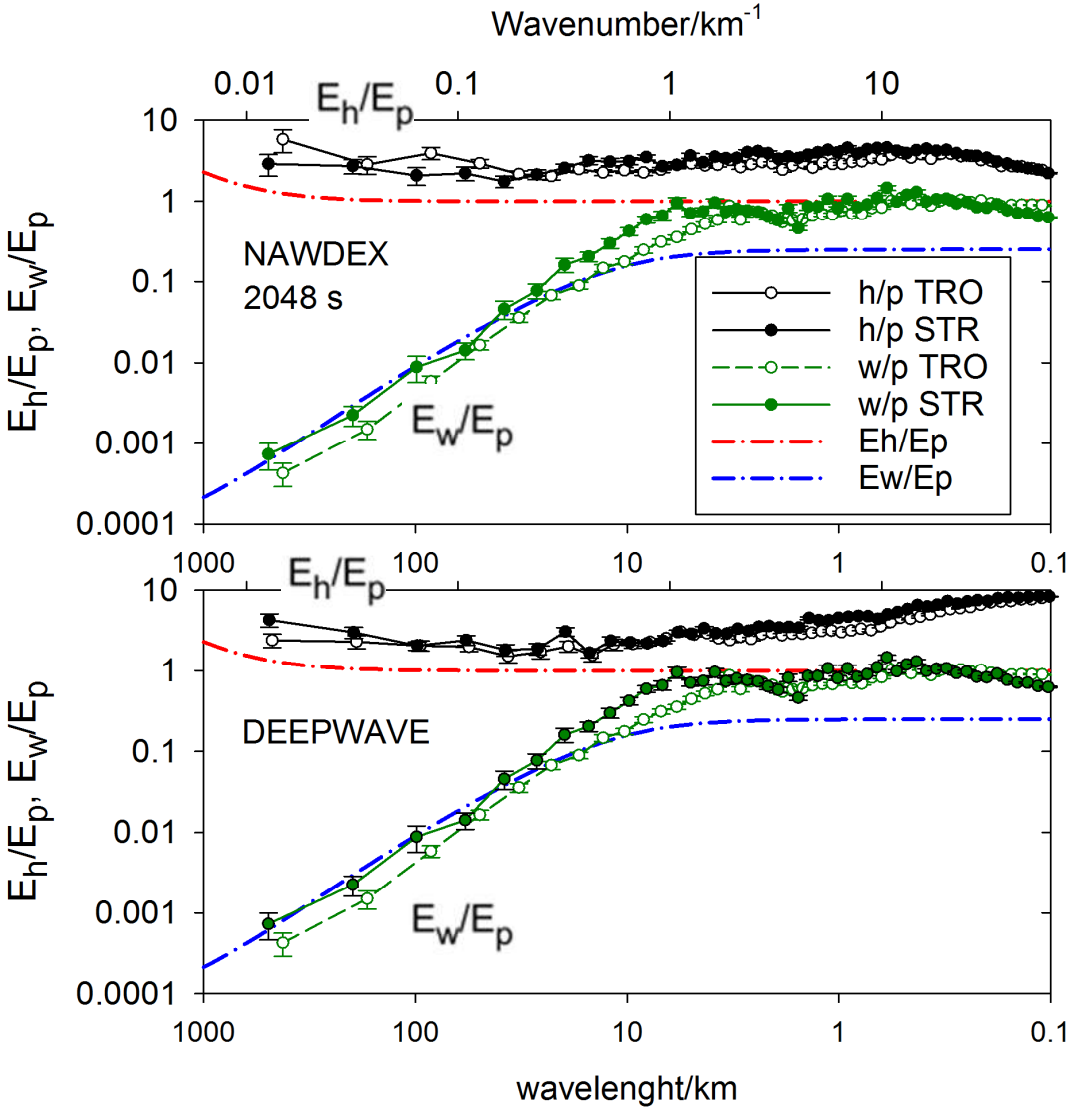


FIG. 3. Variance power spectra of wind and potential temperature near the tropopause from GASP aircraft data. The spectra for meridional wind and temperature are shifted one and two decades to the right, respectively; lines with slopes -3 and $-5/3$ are entered at the same relative coordinates for each variable for comparison.

Ratio of kinetic to potential energies

Comparisons with model spectra reflect contributions from quasi-geostrophic motions and gravity waves

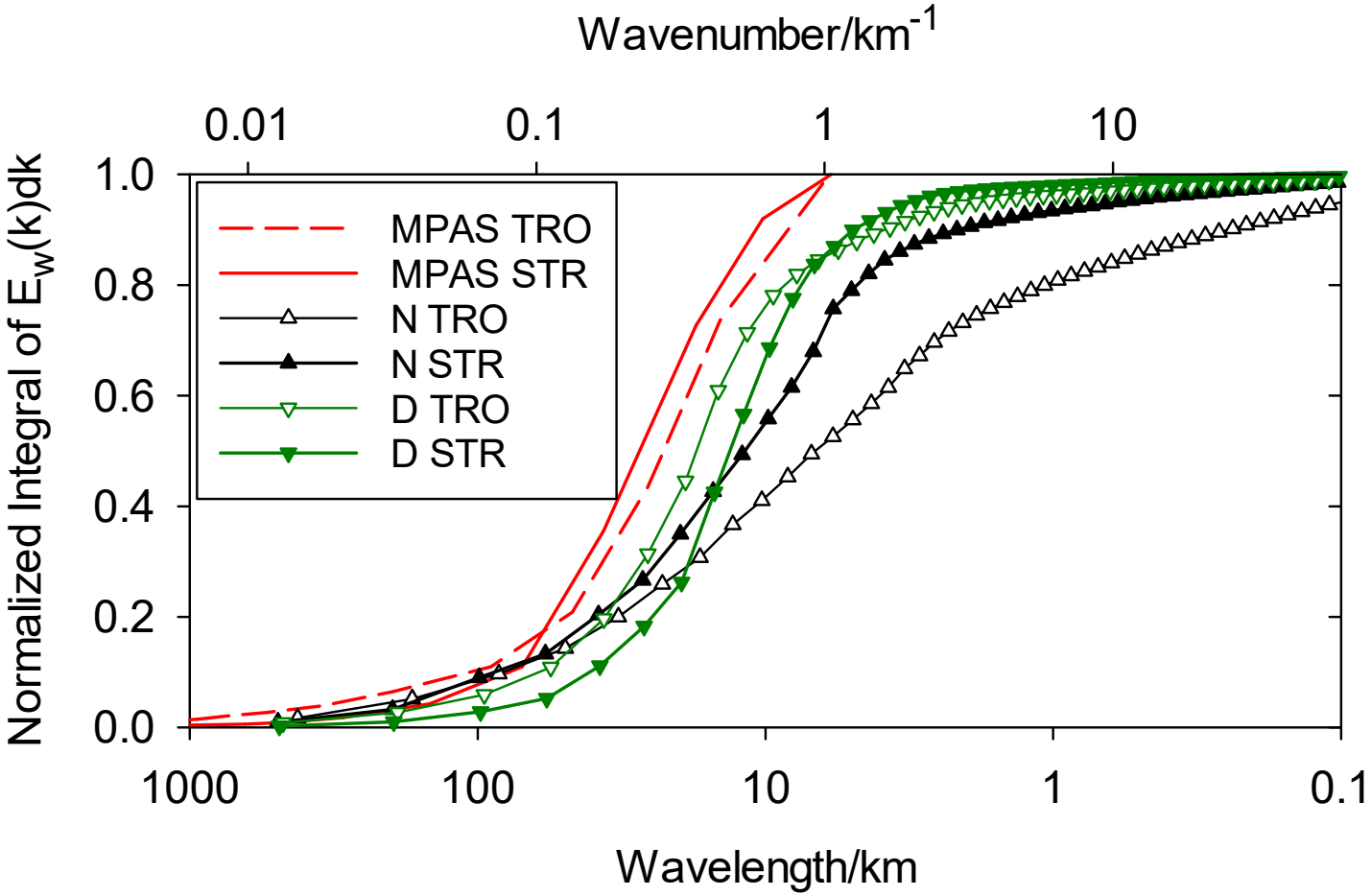


Red and blue lines are models derived from linear gravity theory (e.g., Fritts and Alexander, 2003, Geller and Gong, 2010) for given E_h using our model for E_w/E_h

Consistent with gravity wave theory:
$E_w < E_p < E_h$
for
$f < \omega < N$

But gravity wave models alone underestimate E_h/E_p and E_w/E_p

Which scales contribute most to w-variance?

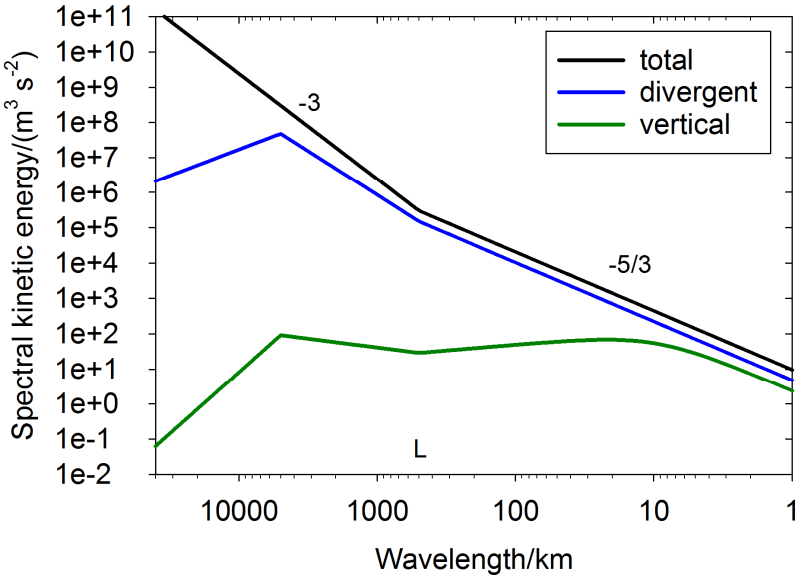


0.5 to 80 km: 90 %
7 to 17 km: 50 %

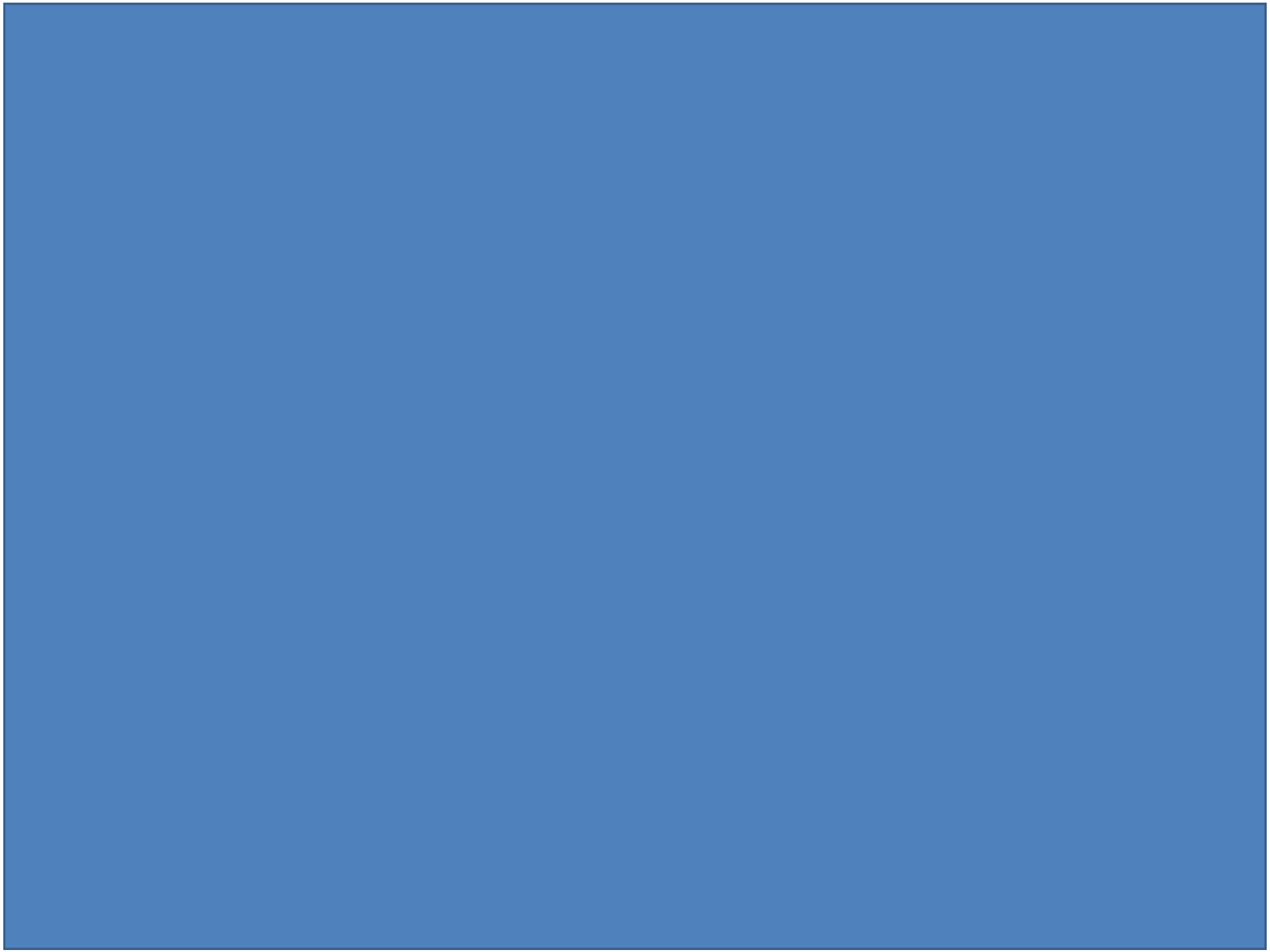
Key Points

- Spectra of vertical and divergent horizontal velocity are connected kinematically at large scales and dynamically at small scales.
- Model is consistent with global (and regional) models and flight experiments within ~30 %
- Maxima in observed vertical velocity spectra near 4 to 15 km wavelengths occur together with flat divergent horizontal spectra.
- Enhanced vertical motions shift the transition scale L between the -3 and a flatter (~-5/3) spectrum in the canonical spectrum to larger scales.

DLR



Enjoy
(e)motions



Measured velocity spectra, mean over all legs